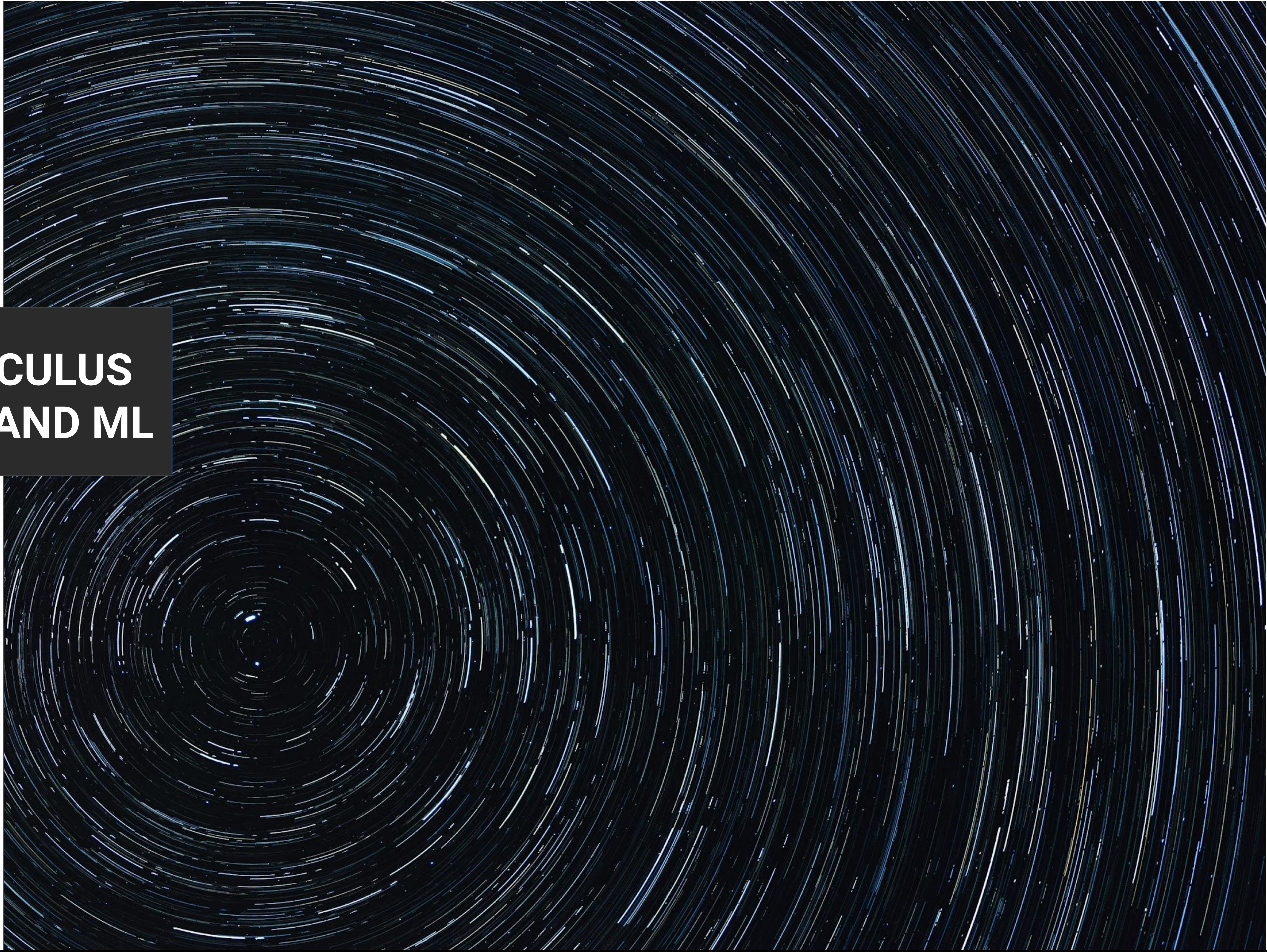
LES FACULTÉS
DE L'UNIVERSITÉ
CATHOLIQUE DE LILLE

Foundations

ELEMENT OF CALCULUS FOR STATISTICS AND ML



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KNOWLEDGE TREE

Part 1: Fundamentals of Calculus

1.1 Introduction to Calculus

- Definition of Limits
- Calculating Limits
- Continuity of Functions

1.2 Derivatives

- Definition of Derivatives
- Rules of Differentiation
- Applications of Derivatives in ML

1.3 Integrals

- Definition of Integrals
- Techniques of Integration
- Calculating Definite Integrals

Part 2: Linear Algebra

2.1 Vectors and Matrices

- Vector Spaces
- Matrix Operations
- Matrix Inversion

2.2 Eigenvalues and Eigenvectors

- Characteristic Equations
- Diagonalization
- Principal Component Analysis (PCA)

2.3 Matrix Calculus

- Gradient Vectors
- Jacobian and Hessian Matrices
- Applications in Optimization

Part 3: Probability and Statistics

3.1 Probability Basics

- Probability Spaces
- Conditional Probability
- Bayes' Theorem

3.2 Probability Distributions

- Discrete and Continuous Distributions
- Gaussian (Normal) Distribution
- Probability Mass and Density Functions

3.3 Statistical Inference

- Hypothesis Testing
- Confidence Intervals
- Maximum Likelihood Estimation

Part 4: Multivariate Calculus and Optimization

4.1 Multivariable Functions

- Partial Derivatives
- Gradient and Hessian Matrix for Multivariable Functions
- Taylor Series Expansion

4.2 Optimization

- Local vs. Global Minima/Maxima
- Gradient Descent
- Constrained Optimization (Lagrange Multipliers)

4.3 Regularization

- L1 and L2 Regularization
- Overfitting and Bias-Variance Trade-off
- Cross-Validation

KEYWORDS (NEW)

Fonction affine | Théorème de dérivation sous le signe d'intégrale | Théorème de Young | Fonction convexe | Fonction exponentielle | Fonction | Fonction hyperbolique

Forme linéaire | Théorème de la règle de la chaîne | Théorème de Parseval | Fonction continue | Fonction linéaire | Fonction polynomiale

Transposée | Théorème de Stone-Weierstrass | Formule de Taylor | Fonction dérivable | Fonction dérivée | Fonction primitive

Fonction réciproque | Forme quadratique | Fonction polynomiale

KEYWORDS

- Calculus
- Mathematics
- Analysis
- Algebra
- Statistics
- Probability
- Optimization
- Machine Learning
- Derivatives
- Integrals
- Matrices
- Vectors
- Probability Distributions
- Statistical Inference
- Regularization
- Vector Spaces
- Gradient
- Hessian
- Multivariate Functions
- Diagonalization
- Principal Component Analysis (PCA)
- Hypothesis Testing
- Confidence Intervals
- Maximum Likelihood Estimation
- Differentiation Rules
- Limit Calculations
- Continuity of Functions
- Integration Techniques
- Local and Global Minima/Maxima
- Gradient Descent
- Constrained Optimization (Lagrange Multipliers)
- L1 and L2 Regularization
- Overfitting
- Bias-Variance Trade-off
- Cross-Validation
- Probability Spaces
- Conditional Probability
- Bayes' Theorem
- Discrete and Continuous Distributions
- Gaussian (Normal) Distribution
- Probability Mass and Density Functions
- Probability Spaces

Use Case with python

Use Case: Optimizing a Linear Regression Using Gradient Descent

In linear regression, one common task is to fit a line (or hyperplane) to data points to minimize the Mean Squared Error (MSE) between the model's predictions and the actual values.

The linear regression model can be formulated as follows:
y=β0+β1x

where y is the dependent variable, **0 β 0** is the intercept, **1 β 1** is the slope of the line, and x is the independent variable.

The goal is to find the values of **0 β 0** and **1 β 1** that minimize the **MSE**.

This can be achieved using gradient descent, an optimization technique that involves computing the partial derivatives of the cost function with respect to 0 β 0 and 1 β 1 and then iteratively updating these parameters until convergence.

In this example, we use the concepts of partial derivatives to compute gradients with respect to 0 β 0 and 1 β 1 , and then iteratively adjust these parameters with a learning rate to minimize the MSE.

This is a simple example to illustrate the use of differential calculus concepts in statistics and ML for model optimization.

In practice, models and cost functions can become much more complex, but the principle remains the same.

Here's a Python code example to optimize a linear regression using gradient descent:

```python
import numpy as np

# Generate random data for the example
np.random.seed(0)
X = 2 * np.random.rand(100, 1)
y = 4 + 3 * X + np.random.rand(100, 1)

# Initialize parameters
beta_0 = np.random.randn()
beta_1 = np.random.randn()

# Hyperparameters
learning_rate = 0.01
n_iterations = 1000

# Gradient descent
for iteration in range(n_iterations):
    y_pred = beta_0 + beta_1 * X
    error = y_pred - y
    gradient_b0 = 2 * np.mean(error)
    gradient_b1 = 2 * np.mean(error * X)
    beta_0 -= learning_rate * gradient_b0
    beta_1 -= learning_rate * gradient_b1

# Display optimized parameters
print("Beta_0 (intercept) =", beta_0)
print("Beta_1 (slope) =", beta_1)
```

REFERENCES V1

Top 15 Libraries in R, Python, MATLAB, or Others for the Given Course Topics

Python Libraries

1. NumPy: For numerical operations, including calculus and linear algebra.
2. SciPy: Extended library built on NumPy, with additional modules for optimization, integration, and other mathematical tasks.
3. TensorFlow : For machine learning, and contains functionalities for derivatives and optimization.
4. SymPy: For symbolic mathematics, including calculus.
5. scikit-learn : Primarily for machine learning but includes several statistical tools.

R Libraries

1. calculus: For symbolic and numerical calculus.
2. pracma: For practical numerical math functions similar to MATLAB.
3. gsl: Wrapper for the GNU Scientific Library, for advanced statistical and mathematical calculations.
4. Matrix: For dense and sparse matrix calculations.
5. mvtnorm: For multivariate normal and t distributions.

MATLAB Toolboxes

1. Symbolic Math Toolbox: For symbolic mathematics.
2. Optimization Toolbox: For optimization problems.
3. Statistics and Machine Learning Toolbox: For statistical analysis and machine learning.
4. Parallel Computing Toolbox: For parallel computation.

Others

1. MathJS (JavaScript): For mathematical operations in web-based applications.

Top 10 Articles or Books

- "**Calculus" by James Stewart**: An excellent introduction to calculus.
- "**Introduction to Linear Algebra**" by Gilbert Strang: For understanding linear algebra concepts.
- "**The Elements of Statistical Learning**" by Hastie, Tibshirani, and Friedman: Machine Learning from a statistical viewpoint.
- "**Probability Theory: The Logic of Science**" by E.T. Jaynes: Covers probability theory deeply.
- "**Convex Optimization**" by Stephen Boyd and Lieven Vandenberghe: Covers optimization extensively.
- "**Deep Learning**" by Ian Goodfellow, Yoshua Bengio, and Aaron Courville: For machine learning aspects.
- "**Numerical Optimization**" by Jorge Nocedal and Stephen J. Wright: Focuses on optimization algorithms and methods.
- "**Pattern Recognition and Machine Learnin**g" by Christopher M. Bishop: Covers statistical methods in ML.
- "**Bayesian Data Analysis**" by Andrew Gelman, John B. Carlin, Hal S. Stern, and Donald B. Rubin: For Bayesian methods.
- "**Multivariate Calculus and Geometry**" by Seán Dineen: For multivariate calculus, particularly relevant to machine learning.

REFERENCES V2

TOP 20 REFERENCES

- Apostol, T. M. (1967). Calculus, Vol. 1: One-Variable Calculus, with an Introduction to Linear Algebra. Wiley.
- Bishop, C. M. (2006). Pattern Recognition and Machine Learning. Springer.
- Casella, G., & Berger, R. L. (2002). Statistical Inference. Duxbury Press.
- Devroye, L., Györfi, L., & Lugosi, G. (1996). A Probabilistic Theory of Pattern Recognition. Springer.
- Goodfellow, I., Bengio, Y., & Courville, A. (2016). Deep Learning. MIT Press.
- Hastie, T., Tibshirani, R., & Friedman, J. (2009). The Elements of Statistical Learning. Springer.
- James, G., Witten, D., Hastie, T., & Tibshirani, R. (2013). An Introduction to Statistical Learning. Springer.
- Keener, R. W. (2011). Theoretical Statistics: Topics for a Core Course. Springer.
- LeCun, Y., Bengio, Y., & Hinton, G. (2015). Deep learning. Nature, 521(7553), 436-444.
- Mohri, M., Rostamizadeh, A., & Talwalkar, A. (2012). Foundations of Machine Learning. MIT Press.
- Murphy, K. P. (2012). Machine Learning: A Probabilistic Perspective. MIT Press.
- Rudin, W. (1976). Principles of Mathematical Analysis. McGraw-Hill.
- Spivak, M. (1967). Calculus. Cambridge University Press.
- Stewart, J. (2015). Calculus: Early Transcendentals. Cengage Learning.
- Strang, G. (1991). Calculus. Wellesley-Cambridge Press.
- Sutton, R. S., & Barto, A. G. (1998). Reinforcement Learning: An Introduction. MIT Press.
- Vapnik, V. N. (1998). Statistical Learning Theory. Wiley.
- Varian, H. R. (2014). Intermediate Microeconomics: A Modern Approach. W. W. Norton & Company.
- Wasserman, L. (2004). All of Statistics: A Concise Course in Statistical Inference. Springer.
- Wierman, A. (2014). Probability and Stochastic Processes. Wiley.

TEXTE DE DESCRIPTION DU COURS

Embark on an enlightening journey through the realm of mathematics as we delve into the fundamentals of Calculus. This immersive experience begins with an enticing narrative, inviting you to explore the captivating world of mathematical analysis and its applications. Imagine yourself as a curious mathematician, poised to unlock the secrets of limits, derivatives, integrals, and more. Calculus isn't just about equations; it's a powerful tool for understanding change, rates, and continuous phenomena in the world around us.

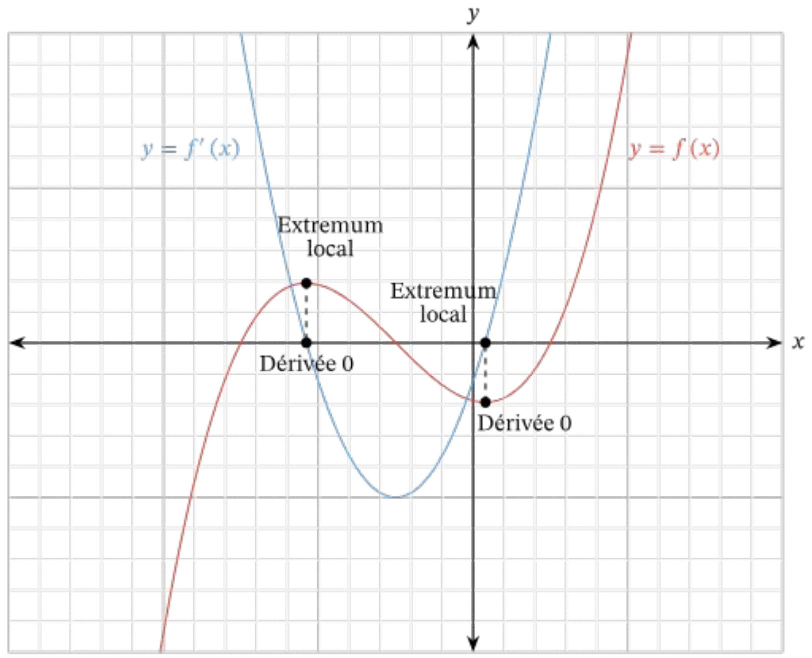
To fully appreciate the significance of Calculus, it's essential to delve into the historical context and the influential mathematicians who have paved the way for this mathematical discipline. Picture luminaries such as Sir Isaac Newton and Gottfried Wilhelm Leibniz, whose pioneering work in the 17th century laid the foundations of Calculus. Their contributions revolutionized mathematics and science, enabling breakthroughs in physics, engineering, and countless other fields. As you embark on this educational journey, you'll follow in the footsteps of these mathematical giants, drawing inspiration from their groundbreaking insights to navigate the complexities of calculus.

The practicality of Calculus extends far beyond the confines of a math classroom. Consider its applications in machine learning, where derivatives are used to optimize models and integrals help calculate areas under curves. Engineers rely on calculus to solve complex problems in structural design, physics, and electrical circuits. Statisticians use probability and statistics, built on calculus principles, to make sense of data and draw meaningful conclusions. Whether you're a student, a data scientist, an engineer, or anyone seeking to harness the power of mathematics in your field, this course offers essential knowledge and skills.

In an age driven by data and scientific inquiry, mastering the fundamentals of Calculus is not just advantageous; it's indispensable. As you progress through this course, you'll gain a deep understanding of mathematical concepts that underpin critical decision-making processes in various disciplines. Calculus empowers you to model the world, analyze complex systems, and make informed predictions. It's a universal language that transcends boundaries and opens doors to innovation. By mastering Calculus, you become a problem solver, a critical thinker, and a contributor to the advancement of science and technology.

Are you ready to embark on this illuminating journey into the world of Calculus? Join us to unlock the full potential of this mathematical tool and elevate your analytical skills. Whether you're driven by academic curiosity, professional growth, or the desire to make a positive impact in your chosen field, this course offers the knowledge and expertise you need. Enroll now to become part of our learning community and begin your exploration of the mathematical wonders that shape our world. Feel free to reach out to the Weeki team for further guidance or any inquiries you may have. Together, let's navigate the intriguing landscape of Calculus and discover the boundless opportunities it presents.

DESIGN DE LA CARD



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Course Name: Simple Linear Regression

#Mathematics
#CalculusBasics
#MathematicalAnalysis