LES FACULTÉS
DE L'UNIVERSITÉ
CATHOLIQUE DE LILLE

Foundations

EPISTEMOLOGY & THEORY OF KNOWLEDGE

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KNOWLEDGE TREE

Part 1: Set Theory and Logic

1.1 Introduction to Sets

- Definition of Sets
- Set Notation
- Set Operations (Union, Intersection, Complement)

1.2 Propositional Logic

- Propositions and Logical Connectives
- Truth Tables
- Logical Implication and Equivalence

1.3 Predicate Logic

- Predicates and Quantifiers (Existential and Universal)
- Translating English Statements into Predicate Logic
- Quantifier Negation and De Morgan's Laws

Part 2: Linear Algebra for Machine Learning

2.1 Vectors and Matrices

- Vector Notation and Operations
- Matrix Notation and Operations
- Matrix-Vector Multiplication

2.2 Eigenvalues and Eigenvectors

- Eigenvalue Equation
- Eigenvector Properties
- Diagonalization of Matrices

2.3 Linear Transformations

- Linear Transformations and Their Properties
- Matrix Representation of Linear Transformations
- Change of Basis

Part 3: Calculus and Optimization

3.1 Limits and Continuity

- Limits of Functions
- Continuity and Discontinuity
- Intermediate Value Theorem

3.2 Derivatives

- Definition of Derivatives
- Rules of Differentiation
- Applications of Derivatives (e.g., Maxima and Minima)

3.3 Integrals

- Indefinite and Definite Integrals
- Fundamental Theorem of Calculus
- Integration Techniques (e.g., Substitution)

Part 4: Probability and Statistics

4.1 Probability Basics

- Sample Spaces and Events
- Probability Distributions
- Conditional Probability and Bayes' Theorem

4.2 Random Variables and Probability Distributions

- Expectation and Variance
- Common Probability Distributions (e.g., Normal, Binomial)
- Probability Density Functions and Cumulative Distribution Functions

4.3 Statistical Inference

- Hypothesis Testing
- Confidence Intervals
- Maximum Likelihood Estimation

KEYWORDS (NEW)

Connaissance · Modèle univarié vs multivarié · Axiome · Déterminisme · Certitude · Darwinisme · Epistémologie

KEYWORDS

- Foundations of Mathematical Thinking
- Mathematics
- Logic
- Set Theory
- Linear Algebra
- Calculus
- Statistics
- Knowledge Acquisition
- Problem Solving
- Confidence
- Precision
- Georg Cantor
- Bertrand Russell
- Mathematical Thought
- Abstract Problems
- Algorithms
- Electrical Circuits
- Control Systems
- Probability
- Inference
- STEM Fields
- Analytical Thinking
- Data Analysis
- Intellectual Exercise
- Problem-Solving Skills
- Data-Driven World
- Advanced Mathematics
- Computer Science
- Scientific Research
- Mathematical Analysis
- Learning Community
- Clarity
- Weeki Team
- Analytical Prowess
- Systematic Understanding

Use Case with python

Use Case: Maximum Likelihood Estimation in Statistical Inference

In the context of the course on Set Theory, Logic, Linear Algebra, Calculus, and Probability and Statistics, let's explore a use case related to statistical inference, specifically Maximum Likelihood Estimation (MLE). MLE is a fundamental concept in statistics that involves finding the parameters of a statistical model that maximize the likelihood of observed data. This use case integrates various mathematical and statistical concepts from the course.

Description:

In this use case, we will focus on applying Maximum Likelihood Estimation to estimate parameters for a probability distribution. MLE is commonly used in machine learning and statistical modeling to find the best-fitting model parameters, such as the mean and variance of a normal distribution.

Key Components:

1. Probability Distributions: Understanding different probability distributions (e.g., normal, binomial) and their probability density functions is crucial for setting up a statistical model.

2. Likelihood Function: The likelihood function expresses how likely the observed data is for various values of the model parameters. It is based on the chosen probability distribution and serves as the foundation for MLE.

3. Derivatives and Calculus: Calculus concepts, such as differentiation, are used to find the maximum of the likelihood function. The first and second derivatives may be involved in optimization.

4. Statistical Inference: MLE is a common method for estimating parameters in statistical inference, which includes hypothesis testing and confidence intervals.

Python Code Example (Maximum Likelihood Estimation):

```python
import numpy as np
from scipy.stats import norm
from scipy.optimize import minimize

# Generate synthetic data from a normal distribution
np.random.seed(0)
data = np.random.normal(loc=3.0, scale=2.0, size=100)

# Likelihood function for a normal distribution
def likelihood(params, data):
    mean, variance = params
    return -np.sum(norm.logpdf(data, loc=mean, scale=np.sqrt(variance)))

# Initial guess for mean and variance
initial_params = [0, 1]

# Find MLE estimates using optimization
result = minimize(likelihood, initial_params, args=(data,))
mle_mean, mle_variance = result.x

print("MLE Estimated Mean:", mle_mean)
print("MLE Estimated Variance:", mle_variance)
```

In this code, we use the likelihood function for a normal distribution to estimate the mean and variance of the distribution that best fits the synthetic data. The scipy.optimize.minimize function is employed to find the parameters that maximize the likelihood.

This use case demonstrates the practical application of mathematical concepts from the course, including probability, calculus, and optimization, in the field of statistical inference using Maximum Likelihood Estimation.

REFERENCES

Top 15 Libraries in R, Python, MATLAB, or Others for the Given Course Topics

Python Libraries

1. NumPy: For numerical operations including linear algebra.
2. SymPy: For symbolic mathematics, including set theory and logic.
3. SciPy: Additional modules for calculus and optimization.
4. TensorFlow: For machine learning, includes functionalities for calculus and optimization.
5. pandas: For data manipulation, helpful in statistical analysis.

R Libraries

1. sets: Provides basic set operations.
2. pracma: Practical numerical math functions.
3. Matrix: For dense and sparse matrix calculations.
4. prob: For probability calculations and simulations.
5. gmodels: Various R programming tools for model fitting.

MATLAB Toolboxes

1. Symbolic Math Toolbox: For symbolic calculations, including set theory and logic.
2. Optimization Toolbox: For optimization problems.
3. Statistics and Machine Learning Toolbox: For statistical analysis and machine learning.

Others

1. MathJS (JavaScript): For mathematical operations.
2. GNU Octave: An alternative to MATLAB for numerical computations.
3. Jupyter Notebook: Useful for mixing code, equations, and text in a single document.
4. Wolfram Mathematica: For symbolic and numerical computations in set theory, logic, calculus, and more.
5. Maple: Another alternative for symbolic math and numerical computations.

Top 10 Articles or Books

- **"Naive Set Theory"** by Paul R. Halmos: For foundational set theory.
- **"Introduction to the Theory of Computation"** by Michael Sipser: For logic and set theory.
- **"Introduction to Linear Algebra" by Gilbert Strang:** For linear algebra.
- **"Calculus"** by James Stewart: Covers calculus extensively.
- **"Convex Optimization"** by Stephen Boyd and Lieven Vandenberghe: For optimization.
- **"A First Course in Probability"** by Sheldon Ross: For probability basics.
- **"The Elements of Statistical Learning"** by Hastie, Tibshirani, and Friedman: For statistical learning.
- **"Statistical Inference"** by George Casella and Roger L. Berger: For statistical inference.
- **"How to Prove It: A Structured Approach"** by Daniel J. Velleman: For logic and proofs.
- **"Pattern Recognition and Machine Learning"** by Christopher M. Bishop: For machine learning aspects with a focus on probability and statistics.

REFERENCES V2

TOP 20 REFERENCES

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TEXTE DE DESCRIPTION DU COURS

Course Description:

Welcome to the captivating journey of "Foundations of Mathematical Thinking." This course invites you to delve into the profound world of mathematical concepts and logical reasoning, setting the stage for a transformative intellectual adventure. Picture yourself stepping into a realm where sets, logic, linear algebra, calculus, and statistics converge to equip you with a robust foundation in mathematical thinking.

As you begin this course, you'll first explore the elegant realm of set theory, where you'll grapple with the fundamental definitions of sets, decipher set notation, and engage in set operations such as union, intersection, and complement. Moving to propositional logic, you'll unravel the intricacies of logical propositions, truth tables, and the nuances of logical implication and equivalence. Predicate logic will introduce you to the power of quantifiers and predicate notation, enabling you to translate complex English statements into precise logical expressions.

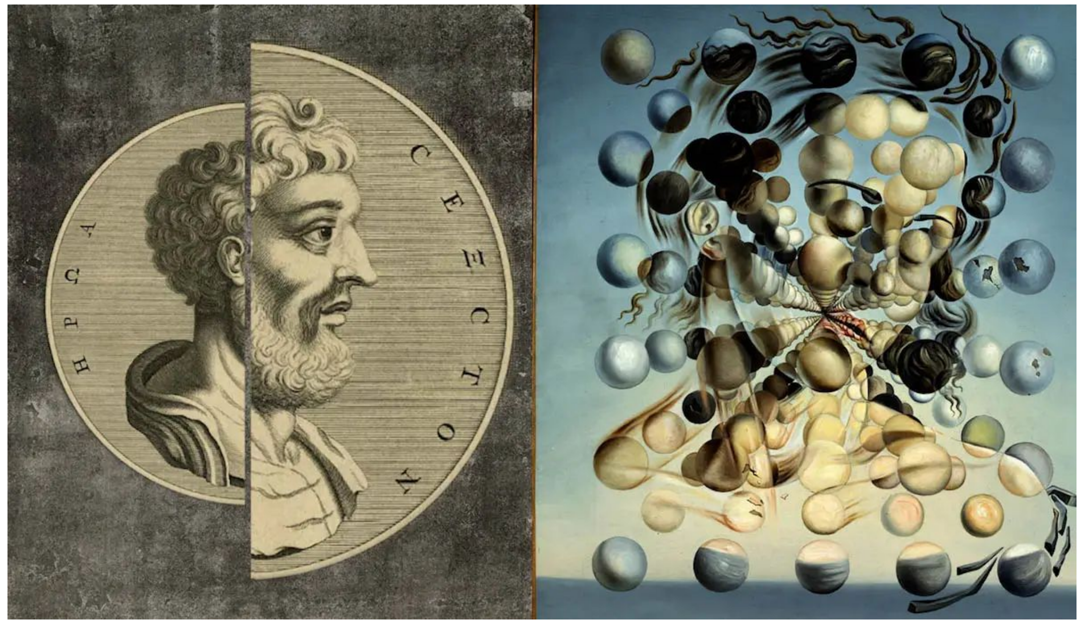
In the second part, you'll journey into the heart of linear algebra for machine learning, a field essential in modern data analysis. You'll master vector and matrix operations, delve into the properties of eigenvalues and eigenvectors, and understand the magic of diagonalization. Linear transformations will become second nature as you explore their properties, matrix representations, and the flexibility of changing bases.

The third part of the course will introduce you to the calculus and optimization, where you'll delve into limits, continuity, and the fascinating Intermediate Value Theorem. Derivatives will be your tool for analyzing the behavior of functions, and you'll learn the rules of differentiation and their applications, including finding maxima and minima. Integral calculus will enable you to calculate areas, solve optimization problems, and understand the fundamental theorem of calculus.

Finally, the course will immerse you in the realm of probability and statistics, essential in making data-driven decisions. You'll explore probability basics, understand different probability distributions, including the ubiquitous normal distribution, and delve into statistical inference, where you'll learn hypothesis testing, confidence intervals, and maximum likelihood estimation.

Whether you're an aspiring mathematician, a data scientist, an engineer, or simply someone captivated by the beauty of mathematical thinking, this course is your gateway to unlocking the language of the universe.

DESIGN DE LA CARD



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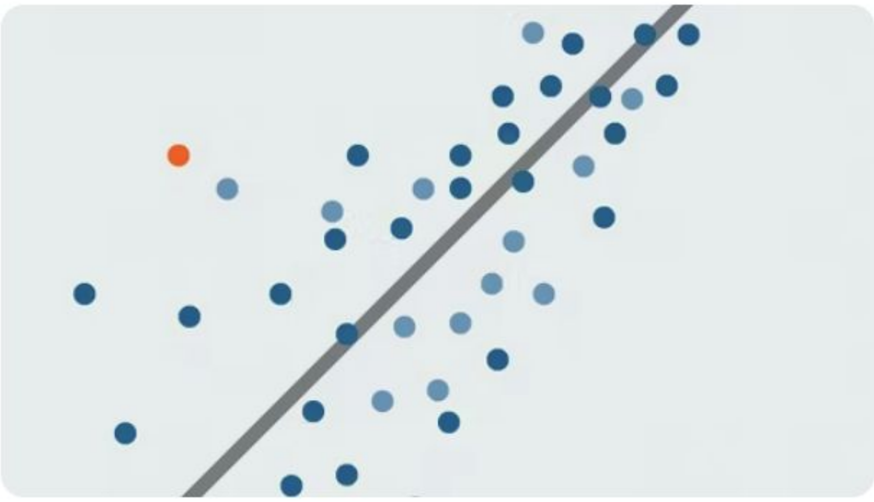
Course Name: Simple Linear Regression

#MathematicalFoundations
#LogicalThinking
#MathematicalConcepts



Duke Duke University

Linear Regression and Modeling

Compétences que vous acquerrez: Probability & Statistics, Regression, Business Analysis, Data Analysis, General Statistics, Statistical Analysis,...

☆ **4.8** (1.7k avis)

Débutant · Course · 1 à 4 semaines