LES FACULTÉS
DE L'UNIVERSITÉ
CATHOLIQUE DE LILLE

Stochastic dynamics & probability

MEASURE THEORY

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KNOWLEDGE TREE

Part 1: Introduction to Measure Theory

1.1 Set Theory Fundamentals

- Definitions of Sets and Elements
- Set Operations (Union, Intersection, Complement)
- Cardinality and Countable Sets

1.2 Real Numbers and Sequences

- Construction of Real Numbers
- Convergence and Limit of Sequences
- Completeness of Real Numbers

1.3 Sigma-Algebras and Measure Spaces

- Definition of Sigma-Algebra
- Measure and Measure Spaces
- Measurable Sets and Functions

Part 2: Lebesgue Measure and Integration

2.1 Lebesgue Measure

- Construction of Lebesgue Measure
- Properties of Lebesgue Measure
- Measurable Sets with Respect to Lebesgue Measure

2.2 Lebesgue Integral

- Definition of Lebesgue Integral
- Properties of Lebesgue Integrals
- Lebesgue Dominated Convergence Theorem

2.3 Convergence in Measure

- Convergence Almost Everywhere
- Convergence in Measure
- Egorov's Theorem

Part 3: Evaluation of Simple Linear Regression

3.1 Probability Spaces

- Probability Measure Definition
- Probability Spaces and Events
- Axioms of Probability

3.2 Probability Distributions

- Discrete and Continuous Distributions
- Probability Density Functions (PDF)
- Cumulative Distribution Functions (CDF)

3.3 Expected Value and Variance

- Definition of Expected Value
- Properties of Expected Value
- Variance and Covariance

Part 4: Applications and Advanced Topics

4.1 Scatterplot

- Absolute Continuity and Singularity
- Radon-Nikodym Theorem Statement
- Applications in Probability Theory

4.2 Measure-Theoretic Probability

- Probability Spaces with Measure Theory
- Laws of Large Numbers
- Central Limit Theorem

4.3 Stochastic Processes

- Introduction to Stochastic Processes
- Markov Chains and Martingales
- Brownian Motion and Wiener Measure

KEYWORDS (NEW)

Espace L^p | Théorème de Radon-Nikodym | Théorème de convergence presque partout | Théorème de convergence uniforme | Théorème de Fubini | Tribu | Mesure spectrale

Théorie des ensembles | Théorème de convergence monotone | Théorème de l'égalité de Jensen | Théorème de Green | Paramètre | Mesure invariante

Théorème de Carathéodory | Convergence en moyenne quadratique | Théorème de Riesz-Markov-Kakutani | Théorème de Jensen | Mesure | Mesure extérieure

Théorème de Cameron-Martin | Théorème de convergence faible | Théorème de Lebesgue | Intégrabilité | Intégrale positive | Fonction mesurable | Mesure de Hausdorff

Théorème de Borel-Cantelli | Théorème de convergence dominée | Théorème de Vitali | Intégrale signée | Intégrale de Henstock-Kurzweil | Théorème de Bernstein | Mesure de Borel

Mesure absolument continue | Théorème de Bolzano-Weierstrass | Théorie de la mesure | Intégrale simple | Intégrale de Lebesgue impropre | Mesure sigma-finie | Espace euclidien

Fonction mesurable négative | Mesure extérieure de Carathéodory | Théorème de Cantor | Mesure atomique | Intégrale de Lebesgue-Stieltjes | Mesure de probabilité | Mesure de Jordan

Fonction mesurable positive | Fonction mesurable essentielle | Théorème de Blackwell | Mesure de Borel | Dérivée radon-nikodym | Mesure de Lebesgue | Mesure de capacité

Espace mesurable | Intégrale de Choquet | Espace de Banach | Ensemble mesurable | Intégrale négative | Intégrale de Pettis | Intégrale de Stieltjes

Fonction caractéristique | Intégrale de Daniell | Espace de Hilbert | Ensemble négligeable | Intégrale de Young | Espace de mesure | Espace des réels | Intégrale de Lebesgue

Fonction de mesure

KEYWORDS

- Measure Theory
- Probability Theory
- Applications
- Probability
- Randomness
- Uncertainty
- Set Theory Fundamentals
- Sets
- Elements
- Set Operations
- Cardinality
- Real Numbers
- Convergence
- Limits of Sequences
- Completeness
- Sigma-Algebras
- Measure Spaces
- Measures
- Measurable Sets
- Measurable Functions
- Mathematical Reasoning
- Lebesgue Measure
- Lebesgue Integrals
- Lebesgue Dominated Convergence Theorem
- Convergence in Measure
- Convergence Almost Everywhere
- Egorov's Theorem
- Probability Measures
- Probability Spaces
- Probability Axioms
- Probability Distributions
- Discrete Distributions
- Continuous Distributions
- Probability Density Functions (PDFs)
- Cumulative Distribution Functions (CDFs)
- Expected Value
- Variance
- Radon-Nikodym Theorem
- Absolute Continuity
- Singularity
- Laws of Large Numbers
- Central Limit Theorem
- Statistical Inference
- Stochastic Processes
- Markov Chains
- Martingales
- Brownian Motion
- Mathematical Foundations
- Data Scientist
- Statistician
- Measure-Theoretic Foundation
- Laws of Large Numbers
- Central Limit Theorem
- Statistical Inference
- Stochastic Processes
- Markov Chains
- Martingales
- Brownian Motion
- Mathematical Foundations
- Data Scientist
- Statistician
- Measure-Theoretic Foundation
- Uncertainty
- Randomness

Use Case with python

Use Case: Probability Distribution Modeling Using Measure Theory

In the context of the course on Measure Theory, Lebesgue Measure and Integration, Probability Measures and Distributions, and Advanced Topics, let's explore a use case related to modeling probability distributions using measure theory. This use case involves the application of measure theory concepts in understanding and modeling probability distributions.

Description:

In this use case, we will focus on modeling a probability distribution, specifically a continuous probability distribution, using the tools and concepts of measure theory. We will use the Lebesgue integral to calculate probabilities and expected values, and explore the properties of probability distributions.

Key Components:

1. Measure Theory Fundamentals: Understanding the basics of sets, sigma-algebras, measures, and measurable sets is essential for modeling probability distributions using measure theory.

2. Lebesgue Measure and Integration: Lebesgue measure and the Lebesgue integral are fundamental concepts for modeling continuous probability distributions and calculating probabilities.

3. Probability Distributions: Understanding discrete and continuous probability distributions, probability density functions (PDFs), and cumulative distribution functions (CDFs) is crucial for modeling real-world phenomena.

4. Expected Value and Variance: Calculating expected values and variances of random variables is an important part of probability distribution modeling.

Python Code Example (Modeling a Continuous Probability Distribution):

```python
import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import norm

# Define parameters of the normal distribution
mu = 0  # Mean
sigma = 1  # Standard deviation

# Generate data points and PDF values for the normal distribution
x = np.linspace(-5, 5, 1000)
pdf_values = norm.pdf(x, loc=mu, scale=sigma)

# Plot the probability density function (PDF) of the normal distribution
plt.figure(figsize=(10, 6))
plt.plot(x, pdf_values, label='Normal PDF', color='blue')
plt.xlabel('Random Variable (x)')
plt.ylabel('Probability Density')
plt.title('Probability Density Function (PDF) of the Normal Distribution')
plt.legend()
plt.grid(True)

# Calculate and print the expected value and variance
expected_value = norm.expect(loc=mu, scale=sigma)
variance = norm.var(loc=mu, scale=sigma)
print(f'Expected Value: {expected_value}')
print(f'Variance: {variance}')

plt.show()
```

In this code, we use the probability density function (PDF) of the normal distribution to model a continuous probability distribution. We calculate and display the PDF, and then calculate the expected value and variance, which are fundamental properties of probability distributions.

This use case demonstrates how measure theory concepts can be applied to model and analyze probability distributions, providing a rigorous foundation for understanding and working with real-world data and phenomena.

TOP 20 REFERENCES

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TEXTE DE DESCRIPTION DU COURS

Welcome to the enlightening journey through "Measure Theory and Probability: Unveiling the Mathematical Foundations." This course invites you to immerse yourself in the profound world of measure theory, probability theory, and their wide-ranging applications. Prepare to embark on an intellectual odyssey that will equip you with the tools to comprehend and navigate the intricacies of probability, randomness, and uncertainty.

As you embark on this course, your voyage begins with a deep dive into set theory fundamentals. You will explore the essence of sets, elements, set operations, and the concept of cardinality. The construction of real numbers and the notion of convergence and limits of sequences will introduce you to the fundamental building blocks of analysis. Completeness of real numbers will emerge as a pivotal concept, bridging the abstract with the concrete.

Sigma-algebras and measure spaces will become your compass as you enter the realm of measure theory. You'll gain a comprehensive understanding of sigma-algebras, measures, and measurable sets and functions, paving the way for precise and rigorous mathematical reasoning.

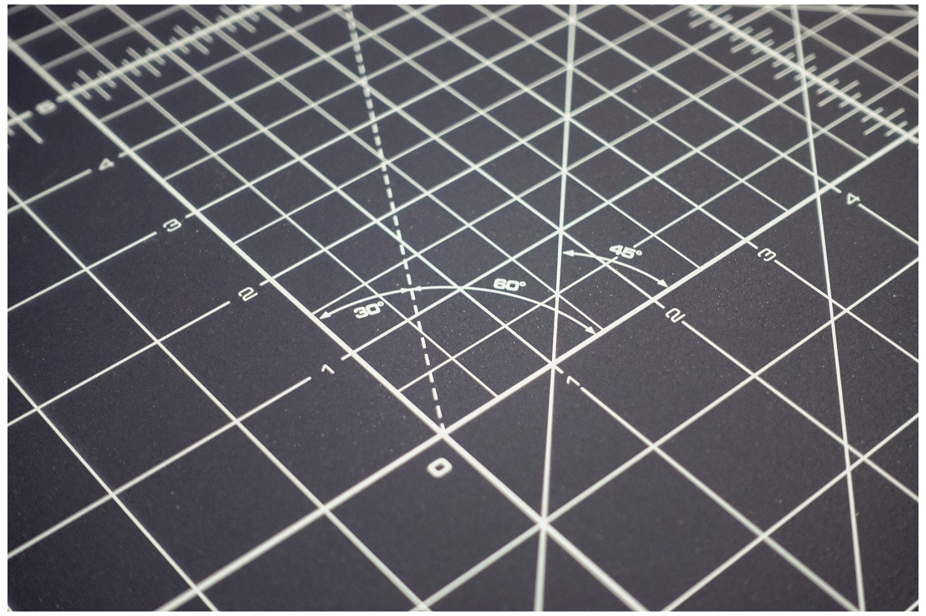
The second part of the course delves into Lebesgue measure and integration, shedding light on the construction and properties of this foundational measure. You'll master the definition of Lebesgue integrals, their properties, and the powerful Lebesgue Dominated Convergence Theorem. Convergence in measure will become second nature, alongside concepts such as convergence almost everywhere and Egorov's Theorem.

The course then transitions seamlessly into probability measures and distributions. Probability spaces will be your foundation, with a clear definition of probability measures and their axioms. You'll delve into probability distributions, distinguishing between discrete and continuous distributions, and gaining insights into probability density functions (PDFs) and cumulative distribution functions (CDFs). Expected value and variance will provide the mathematical tools needed for probabilistic analysis.

In the final part of the course, you'll venture into advanced topics, including the Radon-Nikodym Theorem, which explores absolute continuity and singularity. The measure-theoretic foundation of probability theory will unfold, connecting probability spaces with measure theory. You'll explore the laws of large numbers and the central limit theorem, uncovering the mathematical underpinnings of statistical inference. Stochastic processes will complete your journey, with an introduction to Markov chains, martingales, and the fascinating world of Brownian motion.

Whether you're an aspiring mathematician, a data scientist, a statistician, or simply someone intrigued by the elegant dance between measure theory and probability, this course offers a comprehensive exploration of the mathematical foundations that underpin the world of uncertainty and randomness.

DESIGN DE LA CARD



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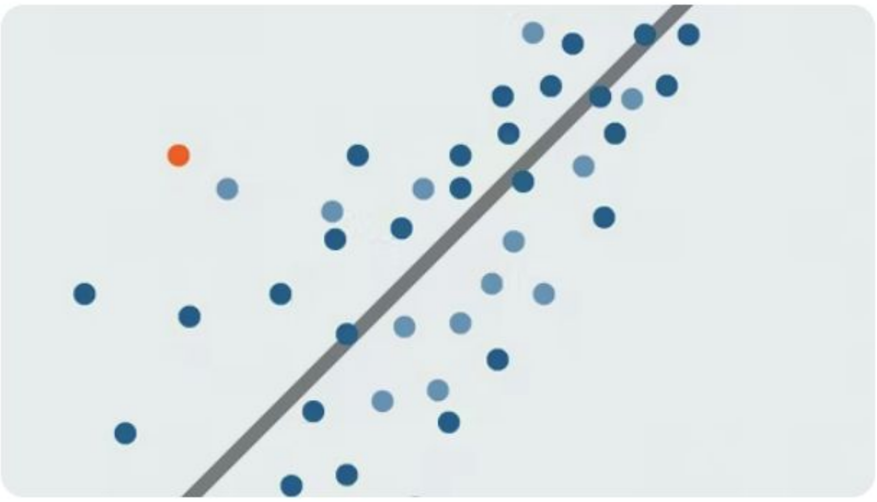
Course Name: Simple Linear Regression

#MeasureTheory
#ProbabilityFoundations
#StochasticProcesses



Duke University

Linear Regression and Modeling

Compétences que vous acquerrez: Probability & Statistics, Regression, Business Analysis, Data Analysis, General Statistics, Statistical Analysis,...

4.8 (1.7k avis)

Débutant · Course · 1 à 4 semaines