LES FACULTÉS
DE L'UNIVERSITÉ
CATHOLIQUE DE LILLE

Models examples

PROBABILITY THEORY

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KNOWLEDGE TREE

Part 1: Foundations of Probability

1.1 Historical and Philosophical Overview

- Origins of probability theory
- Frequentist vs. Bayesian interpretations

1.2 Basic Concepts and Definitions

- Experiment, sample space, events
- Probability axioms

1.3 Rules of Probability

- Addition rule, multiplication rule
- Conditional probability and independence

1.4 Counting Methods

- Permutations and combinations
- Applications in probability

Part 2: Discrete Probability Distributions

2.1 Basics of Discrete Distributions

- Probability mass function (PMF)
- Expected value, variance, and moments

2.2 Binomial and Poisson Distributions

- Assumptions and properties
- Real-world applications and examples

2.3 Hypergeometric and Negative Binomial Distributions

- Differences and similarities to other discrete distributions
- Use cases

2.4 Other Discrete Distributions

- Geometric, multinomial, etc
- Understanding their significance and application

Part 3: Continuous Probability Distributions

3.1 Basics of Continuous Distributions

- Probability density function (PDF) and cumulative distribution function (CDF)
- Expected value, variance, and moments for continuous distributions

3.2 Normal and Exponential Distributions

- Properties and significance
- Central Limit Theorem and its importance

3.3 Beta, Gamma, and Chi-square Distributions

- Relationships between these distributions
- Applications in statistical inference

3.4 Other Continuous Distributions

- Uniform, log-normal, Weibull, etc
- Understanding their context and relevance

Part 4: Advanced Topics in Probability

4.1 Joint and Marginal Distributions

- Bivariate distributions: joint, marginal, and conditional
- Independence and correlation

4.2 Functions of Random Variables

- Transformation techniques
- Moment-generating functions

4.3 Order Statistics

- Distribution of order statistics
- Applications in reliability and life data analysis

4.4 Convergence Concepts in Probability

- Convergence in probability, almost sure convergence
- Law of large numbers and its implications

KNOWLEDGE TREE

KEYWORDS (NEW)

Fonction affine
Loi faible des grands nombres
Théorème de la limite centrale tronquée
Théorie des files d'attente
Théorie des jeux
Fonction
Union (Disjonction)
Vraisemblance
Loi forte des grands nombres
Théorème de convergence en distribution
Système complet d'événements
Variable dépendante
Probabilité
Variable aléatoire
Loi binomiale
Explosion combinatoire
Théorème de convergence en probabilité
Théorème de convergence en loi
Variable indépendante
Kurtosis
Variable endogène
Loi conjointe
Factorisation canonique
Théorème de convergence presque certaine
Théorème de De Moivre-Laplace
Loi des grands nombres
Aléatoire
Variable exogène
Loi marginale
Factorisation de la distribution
Théorème de convergence presque sûre
Fonction de distribution conjointe
Théorème central limite
Variable explicative
Distribution
Fonction de répartition
Théorème de la limite centrale multivariée
Fonction de distribution cumulative
Théorème de Bernoulli
Variable expliquée
Événement
Fonction de vraisemblance
Fonction de densité de probabilité (PDF)
Fonction de masse de probabilité
Théorème de Cochran
Vecteur gaussien
Notion d'ordre
Intersection (Conjonction)
Fréquences relatives cumulées, effectifs cumulés
Formule des probabilités composées
Théorème de Cramér
Fréquence
Convergence
Convergence en probabilité
Convergence essentiellement uniforme
Formule des probabilités totales
Inégalité de Markov
Convergence en loi
Convergence presque sûre
Définition classique des probabilités
Inégalité de Bienaymé-Tchebychev
Log-vraisemblance
Convergence faible
Permutation sans répétition
Espérance mathématique
Événements mutuellement exclusifs
Définition axiomatique de la probabilité
Ensemble négligeable
Convergence forte
Combinatoire analytique
Événement complémentaire
Arrangement avec répétition
Événements incompatibles (disjoints)
Espace d'événements
Espace probabilisé
Événement simple
Événement composé
Combinaison avec répétitions (avec remises)
Événements indépendants
Événement contraire
Notion de répétition
Factoriel d'un entier n
Combinaison sans répétitions (sans remises)
Arrangement sans répétition
Expérience aléatoire
Permutation avec répétition
Axiomes de Kolmogorov
Propriétés des combinaisons, binôme de Newton

ADD TO TREE

- Probability Spaces
- Sigma-Algebras
- Conditional Probabilities
- Random Variables
- Distribution
- Expectation
- Independence
- Generating Functions
- Gaussian Variables
- Convergence

Part 5: Introduction to Probability Spaces

Part 6: Advanced Descriptive Analysis

ADD TO TREE

Part 7: Advanced Probabilistic Formalisms

Part 8: Further Exploration into Probabilistic Formalisms

ADD TO TREE

- ➔ Théorème de la limite centrée
- ➔ Fonction caractéristique
- ➔ Loi normale
- ➔ Illustration numérique
- ➔ Espérance conditionnelle
- ➔ Conditionnement
- ➔ Variable aléatoire
- ➔ Martingale
- ➔ Convergence
- ➔ Temps d'arrêt
- ➔ Marche aléatoire
- ➔ Espaces probabilisés
- ➔ Dénombrement
- ➔ Formule du crible
- ➔ Probabilités conditionnelles
- ➔ Variables aléatoires discrètes
- ➔ Modélisation
- ➔ Variables aléatoires continues
- ➔ Fonctions caractéristiques
- ➔ Théorèmes limites
- ➔ Vecteurs Gaussiens
- ➔ Simulation
- ➔ Estimateurs
- ➔ Tests
- ➔ Intervalles
- ➔ Régions de confiance
- ➔ Contrôles à mi-cours
- ➔ Hardy-Weinberg Model
- ➔ Estimation de la taille d'une population
- ➔ Comparaison de traitements

Part 9: Advanced Probabilistic Concepts and Applications

9.1 Probabilistic Spaces
Enumeration
Sieve Formula and Applications
Conditional Probabilities

9.2 Discrete Random Variables
Exercise Manipulations
Heads or Tails Game
Conditional Laws
Modelling

9.3 Continuous Random Variables
Calculations of Probabilities or Expectations
Law Calculations
Modelling

9.4 Characteristic Functions
Law Calculations
Modelling

9.11 Mid-course Controls
1999-2000: The Collector (I)
2000-2001: The Collector (II)
2001-2002: The Bus Paradox (I)
2002-2003: Mann and Whitney Statistics
2003-2004: Galton Watson Process
2004-2005: Bose-Einstein Law
2005-2006: The Bus Paradox (I)
2006-2007: Polls (I)

9.12 End-of-course Controls
1999-2000: Hardy-Weinberg Model
2000-2001: Estimation of Population Size
2001-2002: Treatment Comparison
2002-2003: Cloud Seeding
2003-2004: Bone Density Comparison
2004-2005: Size of Large Cities
2005-2006: Resistance of Ceramics
2006-2007: Polls (II)

9.13 Corrections
Probabilistic Spaces
Discrete Random Variables
Continuous Random Variables
Characteristic Functions
Limit Theorems
Gaussian Vectors
Simulation
Estimators
Tests
Intervals and Confidence Regions
Mid-course Controls
End-of-course Controls

RAPPELS DE PROBABILITES

Règles opératoires du calcul de l’espérance et de la variance d’une variable aléatoire
Lois de probabilités de variables aléatoires
Vecteurs aléatoires
Vecteurs gaussiens
Théorèmes limites

KEYWORDS

- Probability Theory
- Historical Origins
- Frequentist Interpretation
- Bayesian Interpretation
- Basic Concepts
- Definitions
- Experiments
- Sample Spaces
- Events
- Probability Axioms
- Addition Rule
- Multiplication Rule
- Conditional Probability
- Independence
- Counting Methods
- Permutations
- Combinations
- Probability Mass Functions (PMFs)
- Expected Values
- Variances
- Moments
- Binomial Distribution
- Poisson Distribution
- Hypergeometric Distribution
- Negative Binomial Distribution
- Geometric Distribution
- Multinomial Distribution
- Continuous Probability Distributions
- Probability Density Functions (PDFs)
- Cumulative Distribution Functions (CDFs)
- Normal Distribution
- Exponential Distribution
- Central Limit Theorem
- Beta Distribution
- Gamma Distribution
- Chi-Square Distribution
- Uniform Distribution
- Log-Normal Distribution
- Weibull Distribution
- Advanced Probability Topics
- Joint and Marginal Distributions
- Bivariate Distributions
- Independence
- Correlation
- Functions of Random Variables
- Transformation Techniques
- Moment-Generating Functions
- Order Statistics
- Reliability
- Life Data Analysis
- Convergence in Probability
- Almost Sure Convergence
- Law of Large Numbers
- Statistician
- Data Scientist
- Uncertainty
- Confidence
- Expertise
- Knowledge

Use Case: Analyzing Risk with Probability Distributions

In the context of the course on Foundations of Probability, Discrete and Continuous Probability Distributions, and Advanced Topics in Probability, let's explore a use case related to analyzing risk using probability distributions. This use case involves the application of probability concepts to assess and manage risk in financial investments.

Description:

In this use case, we will focus on modeling the risk associated with a financial investment using probability distributions. We will consider a hypothetical investment in a stock and assess the potential returns and risks associated with it by modeling the stock's price movement.

Key Components:

1. Foundations of Probability: Understanding the basic concepts of probability, such as sample space, events, and probability axioms, is essential for modeling and assessing risk.

2. Discrete and Continuous Probability Distributions: Knowledge of discrete and continuous probability distributions, including their probability mass functions (PMFs) and probability density functions (PDFs), is crucial for modeling financial data.

3. Expected Value and Variance: Calculating the expected value and variance of a random variable helps assess the average return and risk associated with an investment.

4. Advanced Topics in Probability: Concepts such as joint and marginal distributions, functions of random variables, and convergence concepts are useful for analyzing the relationship between multiple financial instruments.

Python Code Example (Analyzing Investment Risk):

```python
import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import norm

# Define parameters of the stock's return distribution
mean_return = 0.08  # Mean return
volatility = 0.15  # Annual volatility (standard deviation)

# Generate a range of possible annual returns
returns = np.linspace(-0.3, 0.3, 1000)

# Calculate the probability density function (PDF) using the normal distribution
pdf_values = norm.pdf(returns, loc=mean_return, scale=volatility)

# Plot the PDF of the stock's returns
plt.figure(figsize=(10, 6))
plt.plot(returns, pdf_values, label='PDF', color='blue')
plt.xlabel('Annual Return')
plt.ylabel('Probability Density')
plt.title('Probability Density Function (PDF) of Stock Returns')
plt.legend()
plt.grid(True)

# Calculate and print the expected return and risk (standard deviation)
expected_return = mean_return
risk = volatility
print(f'Expected Return: {expected_return:.2%}')
print(f'Risk (Standard Deviation): {risk:.2%}')

plt.show()
```

In this code, we model the probability distribution of a stock's annual returns using a normal distribution with a specified mean return and volatility. We calculate and display the probability density function (PDF) of returns and then calculate the expected return and risk (standard deviation) as measures of investment risk.

This use case demonstrates how probability concepts can be applied to assess investment risk, allowing investors and analysts to make informed decisions based on the expected return and risk associated with a financial instrument.

TOP 20 REFERENCES

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TEXTE DE DESCRIPTION DU COURS

Welcome to "Probability Unveiled: Exploring the Foundations and Beyond." This course is your gateway to unraveling the fascinating world of probability theory from its historical origins to its advanced applications in modern statistics and data science. Prepare to embark on a captivating journey through the fundamental principles, discrete and continuous probability distributions, and advanced probability concepts.

As your voyage begins, we'll take you on a historical and philosophical overview of probability theory. You'll discover the intriguing origins of probability theory and explore the timeless debate between frequentist and Bayesian interpretations. These philosophical underpinnings will provide you with a profound perspective on probability.

In the section on basic concepts and definitions, you'll delve into the core building blocks of probability theory. Learn to conceptualize experiments, sample spaces, and events, and gain a solid grasp of the probability axioms that form the bedrock of probability theory. Rules of probability, including the addition and multiplication rules, as well as conditional probability and independence, will become your second nature.

Counting methods will equip you with powerful tools for probability analysis. Understand permutations and combinations and witness their real-world applications in solving intricate probability problems.

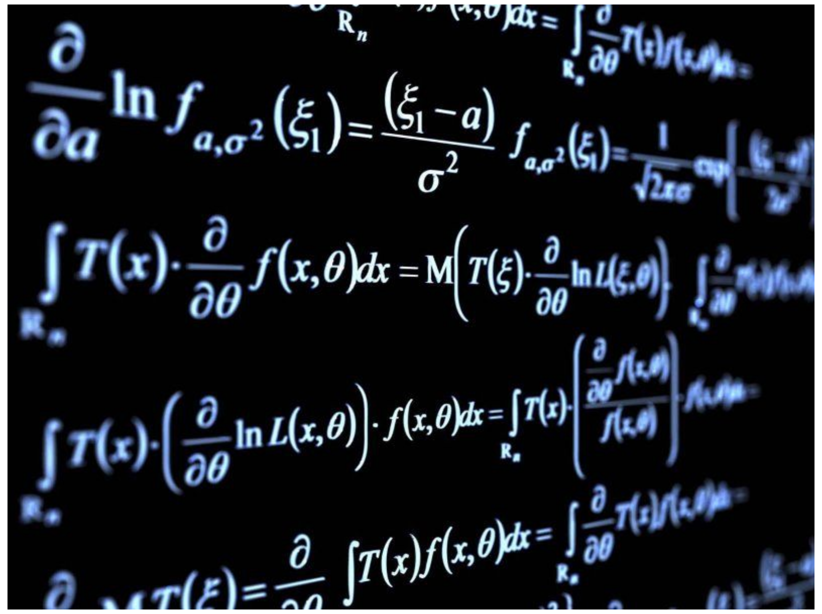
The course then transitions into discrete probability distributions, where you'll explore the basics of probability mass functions (PMFs) and delve into expected values, variances, and moments. Dive deep into the binomial and Poisson distributions, examining their assumptions, properties, and practical applications. Discover the hypergeometric and negative binomial distributions and learn how to wield them effectively in various scenarios. Gain insights into other discrete distributions such as the geometric and multinomial distributions, and understand their significance.

Continuing your journey, you'll explore continuous probability distributions. Get comfortable with probability density functions (PDFs) and cumulative distribution functions (CDFs), and master the calculations of expected values, variances, and moments for continuous distributions. The normal and exponential distributions will take center stage, revealing their properties and significance, as well as their role in the Central Limit Theorem. Delve into the beta, gamma, and chi-square distributions, unraveling their relationships and their applications in statistical inference. Explore other continuous distributions like the uniform, log-normal, and Weibull distributions, gaining insights into their contexts and relevance.

The final part of the course introduces advanced topics in probability. Understand the intricacies of joint and marginal distributions, exploring bivariate distributions, independence, and correlation. Master functions of random variables, including transformation techniques and moment-generating functions. Explore the distribution of order statistics and its applications in reliability and life data analysis. Finally, grasp the concept of convergence in probability and almost sure convergence, and discover the profound implications of the law of large numbers.

Whether you're a budding statistician, a data scientist, or simply someone intrigued by the art of uncertainty, this course will equip you with the knowledge and tools to navigate the intricate world of probability with confidence and expertise.

DESIGN DE LA CARD



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Course Name: Simple Linear Regression

#ProbabilityFoundations
#ProbabilityDistributions
#AdvancedProbability