



LES FACULTÉS  
DE L'UNIVERSITÉ  
CATHOLIQUE DE LILLE

Models examples

## PROBABILITY THEORY

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# KNOWLEDGE TREE

## Part 1: Foundations of Probability

### 1.1 Historical and Philosophical Overview

- Origins of probability theory
- Frequentist vs. Bayesian interpretations

### 1.2 Basic Concepts and Definitions

- Experiment, sample space, events
- Probability axioms

### 1.3 Rules of Probability

- Addition rule, multiplication rule
- Conditional probability and independence

### 1.4 Counting Methods

- Permutations and combinations
- Applications in probability

## Part 2: Discrete Probability Distributions

### 2.1 Basics of Discrete Distributions

- Probability mass function (PMF)
- Expected value, variance, and moments

### 2.2 Binomial and Poisson Distributions

- Assumptions and properties
- Real-world applications and examples

### 2.3 Hypergeometric and Negative Binomial Distributions

- Differences and similarities to other discrete distributions
- Use cases

### 2.4 Other Discrete Distributions

- Geometric, multinomial, etc
- Understanding their significance and application

## Part 3: Continuous Probability Distributions

### 3.1 Basics of Continuous Distributions

- Probability density function (PDF) and cumulative distribution function (CDF)
- Expected value, variance, and moments for continuous distributions

### 3.2 Normal and Exponential Distributions

- Properties and significance
- Central Limit Theorem and its importance

### 3.3 Beta, Gamma, and Chi-square Distributions

- Relationships between these distributions
- Applications in statistical inference

### 3.4 Other Continuous Distributions

- Uniform, log-normal, Weibull, etc
- Understanding their context and relevance

## Part 4: Advanced Topics in Probability

### 4.1 Joint and Marginal Distributions

- Bivariate distributions: joint, marginal, and conditional
- Independence and correlation

### 4.2 Functions of Random Variables

- Transformation techniques
- Moment-generating functions

### 4.3 Order Statistics

- Distribution of order statistics
- Applications in reliability and life data analysis

### 4.4 Convergence Concepts in Probability

- Convergence in probability, almost sure convergence
- Law of large numbers and its implications

# KNOWLEDGE TREE

Introduction aux espaces probabilisés

- 1.2 Espaces probabilisés - Tribus . . .
- 1.2 Espaces probabilisés - Probabilité
- 1.3 Espaces probabilisés finis
- 1.4 Construction d'espaces probabilisés généraux
- 1.5 Probabilités conditionnelles
- 1.6 Indépendance de tribus et d'événements

2 Variables aléatoires - 2.1 Définition

2 Variables aléatoires - Loi, espérance - 2.2.1 Loi d'une variable aléatoire réelle, fonction de répartition

2 Variables aléatoires - Loi, espérance - Espérance d'une variable aléatoire réelle

2 Variables aléatoires - Loi, espérance - 2.2.3 Variables aléatoires réelles discrètes

2 Variables aléatoires - Loi, espérance - 2.2.4 Variables aléatoires réelles à densité.

2 Variables aléatoires - Loi, espérance - 2.2.5 Variables aléatoires vectorielles

2.3 Indépendance de variables aléatoires

2.4 Somme de variables aléatoires indépendantes, produit de convolution

2.5 Moments d'une variable aléatoire réelle, variance, écart type

2.6 Covariance, coefficient de corrélation

3 Fonctions génératrices, fonctions caractéristiques, variables gaussiennes 35 - 3.1 Fonctions génératrices

3 Fonctions génératrices, fonctions caractéristiques, variables gaussiennes 35 - 3.2 Fonctions caractéristiques

3 Fonctions génératrices, fonctions caractéristiques, variables gaussiennes 35 - 3.3 Variables aléatoires gaussiennes

Convergence des suites de variables aléatoires, loi des grands nombres 4.1 Convergence des suites de variables aléatoires 4.1.1 Lemme de Borel-Cantelli

Convergence des suites de variables aléatoires, loi des grands nombres 4.1 Convergence des suites de variables aléatoires4.1.2 Inégalités . . . . .

Convergence des suites de variables aléatoires, loi des grands nombres 4.1 Convergence des suites de variables aléatoires4.1.3 Convergence presque sûre

Convergence des suites de variables aléatoires, loi des grands nombres 4.1 Convergence des suites de variables aléatoires4.1.4 Convergence dans LP

Convergence des suites de variables aléatoires, loi des grands nombres 4.1 Convergence des suites de variables aléatoires4.1.5 Convergence en probabilité. . . . .

Convergence des suites de variables aléatoires, loi des grands nombres 4.1 Convergence des suites de variables aléatoires Comparaison des différents types de convergence

Convergence des suites de variables aléatoires, loi des grands nombres Loi forte des grands nombres

- 4.3 Applications de la loi des grands nombres . . .
- 4.4 Tribu asymptotique, loi du 0-1 de Kolmogorov
- 4.5 Compléments sur la convergence en probabilité.

5 Convergence en loi, théorème limite central - 5.1 Convergence des mesures bornées sur Rd

5 Convergence en loi, théorème limite central - 5.1 Convergence en loi

5 Convergence en loi, théorème limite central - 5.3 Convergence en loi et convergence des fonctions caractéristiques

5 Convergence en loi, théorème limite central - 5.4 Théorème limite central . . . . .

5 Convergence en loi, théorème limite central- Loi du x?, théorème de Karl Pearson

# KEYWORDS (NEW)

Fonction affine	Loi faible des grands nombres	Théorème de la limite centrale tronquée	Théorie des files d'attente	Théorie des jeux	Fonction	Union (Disjonction)
Vraisemblance	Loi forte des grands nombres	Théorème de convergence en distribution	Système complet d'événements	Variable dépendante	Probabilité	Variable aléatoire
Loi binomiale	Explosion combinatoire	Théorème de convergence en probabilité	Théorème de convergence en loi	Variable indépendante	Kurtosis	Variable endogène
Loi conjointe	Factorisation canonique	Théorème de convergence presque certaine	Théorème de De Moivre-Laplace	Loi des grands nombres	Aléatoire	Variable exogène
Loi marginale	Factorisation de la distribution	Théorème de convergence presque sûre	Fonction de distribution conjointe	Théorème central limite	Variable explicative	
Distribution	Fonction de répartition	Théorème de la limite centrale multivariée	Fonction de distribution cumulative	Théorème de Bernoulli	Variable expliquée	
Événement	Fonction de vraisemblance	Fonction de densité de probabilité (PDF)	Fonction de masse de probabilité	Théorème de Cochran	Vecteur gaussien	
Notion d'ordre	Intersection (Conjonction)	Fréquences relatives cumulées, effectifs cumulés	Formule des probabilités composées	Théorème de Cramér	Fréquence	
Convergence	Convergence en probabilité	Convergence essentiellement uniforme	Formule des probabilités totales	Inégalité de Markov	Convergence en loi	
Convergence presque sûre	Définition classique des probabilités	Inégalité de Bienaymé-Tchebychev	Log-vraisemblance	Convergence faible	Permutation sans répétition	
Espérance mathématique	Événements mutuellement exclusifs	Définition axiomatique de la probabilité	Ensemble négligeable	Convergence forte	Combinatoire analytique	
Événement complémentaire	Arrangement avec répétition	Événements incompatibles (disjoints)	Espace d'événements	Espace probabilisé	Événement simple	
Événement composé	Combinaison avec répétitions (avec remises)	Événements indépendants	Événement contraire	Notion de répétition	Factoriel d'un entier n	
Combinaison sans répétitions (sans remises)	Arrangement sans répétition	Expérience aléatoire	Permutation avec répétition	Axiomes de Kolmogorov		
Propriétés des combinaisons, binôme de Newton						

# ADD TO TREE

Probability Spaces

Sigma-Algebras

Conditional Probabilities

Random Variables

Distribution

Expectation

Independence

Generating Functions

Gaussian Variables

Convergence

## Part 5: Introduction to Probability Spaces

### 5.1 Introduction

- Overview

### 5.2 Probability Spaces

- Sigma-Algebras
- Probability

### 5.3 Finite Probability Spaces

- Overview of Finite Probability Spaces

### 5.4 Construction of General Probability Spaces

- Overview of General Probability Space Construction

### 5.5 Conditional Probabilities

- Overview of Conditional Probabilities

### 5.6 Independence of Sigma-Algebras and Events

- Overview of Independence Concepts

### 5.7 Random Variables

- Definition
- Distribution, Expectation
- Distribution of a Real Random Variable, Cumulative Distribution Function
- Expectation of a Real Random Variable
- Discrete Real Random Variables
- Density Real Random Variables
- Vectorial Random Variables
- Independence of Random Variables
- Sum of Independent Random Variables, Convolution Product
- Moments of a Real Random Variable, Variance, Standard Deviation
- Covariance, Correlation Coefficient

## 5.8 Generating Functions, Characteristic Functions, Gaussian Variables

- Generating Functions
- Characteristic Functions
- Gaussian Random Variables

## 5.9 Convergence of Sequences of Random Variables, Law of Large Numbers

- Convergence of Sequences of Random Variables
- Borel-Cantelli Lemma
- Inequalities
- Almost Sure Convergence
- Convergence in LP
- Convergence in Probability
- Comparison of Different Types of Convergence
- Strong Law of Large Numbers

## Part 6: Advanced Descriptive Analysis

### 6.1 Description of a Variable

- Measurement Process
- Structure of Measurement
- Nominal Qualitative Variable
- Ordinal Qualitative Variable
- Discrete Quantitative Variable
- Continuous Quantitative Variable
- Concept of Ratio and Interval
- Summary

### 6.2 Description of Relationship

- Relationship Between a Numeric Variable and a Categorized Variable
- Analysis Plans
- Comparison of Centrality Indices
- Comparison of Dispersions
- Block Distribution Comparison
- Locating an Individual
- Relationship Between Two Numeric Variables
- Empirical Covariance
- Bravais-Pearson Correlation Coefficient
- Correlation and Causality
- Relationship Between Two Categorized Variables
- Odds, Odds Ratios and Log Odds Ratios
- Likelihood Ratio

## 6.3 Algebra of Events

- Concept of Set
- Intersection and Union
- Algebra on Sets
- Order of Operators
- Distributivity
- De Morgan's Laws
- Summary Table
- Application: Card Selection Game

## 6.4 Probability Calculations

- Intuitive Concept
- Known Probability
- Unknown Probability
- Joint, Conditional, and Marginal Probability
- Bayes' Formulas
- Product Rule
- Addition Rule
- Law of Total Probability
- Summary Table
- Counting
- Permutations
- Arrangements
- Combinations
- Distribution into Identified Classes
- Summary Table
- Probabilities on a Non-countable Set
- Simulation of a Uniform Process
- Point Probability in a Continuous Law
- Construction of Uniform Density
- Concept of Integral
- Applications
- Is Sally Clark Guilty?

## ADD TO TREE

### Part 7: Advanced Probabilistic Formalisms

#### 7.1 Introduction to Kolmogorov Formalism

Discrete Case: Finite or Countable

Continuous Case:  $S = R$  or  $R^*$

Product Spaces Case

Exercises

#### 7.2 Random Variables

Definition of a Random Variable

Expectation and Variance

Inequalities

Law of a Random Variable

Law of a Tuple of Random Variables

Exercises

#### 7.3 Independence

Independence of Events and Random Variables

Borel-Cantelli Lemma

Law of a Tuple of Independent Variables

Exercises

#### 7.4 Law of Large Numbers

Weak Law of Large Numbers

Strong Law of Large Numbers

Numerical Illustration

Exercises

#### 7.5 Convergence of Sequences of Random Variables

Different Types of Convergence

Characteristic Function and Fourier Transform

Convergence in Law

Exercises

#### 7.6 Central Limit Theorem

Characteristic Function of the Normal Distribution

Central Limit Theorem

Numerical Illustration

Exercises

### 7.7 Conditional Expectation

Definition of Conditional Expectation

Examples

Properties of Conditional Expectation

Conditioning Relative to a Random Variable

Exercises

### 7.8 Martingale Theory

Concept of Martingale

Convergence of Martingales

Series of Independent Random Variables

Convergence of Conditional Expectations

Stopping Time

Illustration with a Random Walk

Exercises

### 7.A Integration Recap

Convergence Theorems

Integrals Depending on a Parameter

Multiple Integrals

$L^p$  Spaces

Inequalities

Fourier Inversion Formula

Exercises and Counterexamples

### 7.B Formula Sheet

Law of a Random Variable

Inequalities

Pairs of Random Variables

Convergence of Random Variables

Limit Theorems

Conditional Expectation

Martingales

### 7.C References

### Part 8: Further Exploration into Probabilistic

#### Formalisms

#### 8.1 Kolmogorov Formalism

Discrete Case: Finite or Countable

Continuous Case:  $S = R$  or  $R^*$

Product Spaces Case

Exercises

#### 8.2 Random Variables

Definition of a Random Variable

Expectation and Variance

Inequalities

Law of a Random Variable

Law of a Tuple of Random Variables

Exercises

#### 8.3 Independence

Independence of Events and Random Variables

Borel-Cantelli Lemma

Law of a Tuple of Independent Variables

Exercises

#### 8.4 Law of Large Numbers

Weak Law of Large Numbers

Strong Law of Large Numbers

Numerical Illustration

Exercises

#### 8.5 Convergence of Sequences of Random Variables

Different Types of Convergence

Characteristic Function and Fourier Transform

Convergence in Law

Exercises

#### 8.B Formula Sheet

Law of a Random Variable

Inequalities

Pairs of Random Variables

Convergence of Random Variables

Limit Theorems

Conditional Expectation

Martingales

### 8.6 Central Limit Theorem

Characteristic Function of the Normal Distribution

Central Limit Theorem

Numerical Illustration

Exercises

### 8.7 Conditional Expectation

Definition of Conditional Expectation

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Properties of Conditional Expectation

Conditioning Relative to a Random Variable

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### 8.8 Martingale Theory

Concept of Martingale

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### 8.A Integration Recap

Convergence Theorems

Integrals Depending on a Parameter

Multiple Integrals

$L^p$  Spaces

Inequalities

Fourier Inversion Formula

Exercises and Counterexamples

### 8.C References

## ADD TO TREE

- Théorème de la limite centrée
- Fonction caractéristique
- Loi normale
- Illustration numérique
- Espérance conditionnelle
- Conditionnement
- Variable aléatoire
- Martingale
- Convergence
- Temps d'arrêt
- Marche aléatoire
- Espaces probabilisés
- Dénombrement
- Formule du crible
- Probabilités conditionnelles
- Variables aléatoires discrètes
- Modélisation
- Variables aléatoires continues
- Fonctions caractéristiques
- Théorèmes limites
- Vecteurs Gaussiens
- Simulation
- Estimateurs
- Tests
- Intervalles
- Régions de confiance
- Contrôles à mi-cours
- Hardy-Weinberg Model
- Estimation de la taille d'une population
- Comparaison de traitements

### Part 9: Advanced Probabilistic Concepts and Applications

- 9.1 Probabilistic Spaces
  - Enumeration
  - Sieve Formula and Applications
  - Conditional Probabilities
- 9.2 Discrete Random Variables
  - Exercise Manipulations
  - Heads or Tails Game
  - Conditional Laws
  - Modelling
- 9.3 Continuous Random Variables
  - Calculations of Probabilities or Expectations
  - Law Calculations
  - Modelling
- 9.4 Characteristic Functions
  - Law Calculations
  - Modelling
- 9.11 Mid-course Controls
  - 1999-2000: The Collector (I)
  - 2000-2001: The Collector (II)
  - 2001-2002: The Bus Paradox (I)
  - 2002-2003: Mann and Whitney Statistics
  - 2003-2004: Galton Watson Process
  - 2004-2005: Bose-Einstein Law
  - 2005-2006: The Bus Paradox (I)
  - 2006-2007: Polls (I)
- 9.12 End-of-course Controls
  - 1999-2000: Hardy-Weinberg Model
  - 2000-2001: Estimation of Population Size
  - 2001-2002: Treatment Comparison
  - 2002-2003: Cloud Seeding
  - 2003-2004: Bone Density Comparison
  - 2004-2005: Size of Large Cities
  - 2005-2006: Resistance of Ceramics
  - 2006-2007: Polls (II)
- 9.13 Corrections
  - Probabilistic Spaces
  - Discrete Random Variables
  - Continuous Random Variables
  - Characteristic Functions
  - Limit Theorems
  - Gaussian Vectors
  - Simulation
  - Estimators
  - Tests
  - Intervals and Confidence Regions
  - Mid-course Controls
  - End-of-course Controls

### RAPPELS DE PROBABILITES

- Règles opératoires du calcul de l'espérance et de la variance d'une variable aléatoire
- Lois de probabilités de variables aléatoires
- Vecteurs aléatoires
- Vecteurs gaussiens
- Théorèmes limites

## KEYWORDS

- Probability Theory
- Historical Origins
- Frequentist Interpretation
- Bayesian Interpretation
- Basic Concepts
- Definitions
- Experiments
- Sample Spaces
- Events
- Probability Axioms
- Addition Rule
- Multiplication Rule
- Conditional Probability
- Independence
- Counting Methods
- Permutations
- Combinations
- Probability Mass Functions (PMFs)
- Expected Values
- Variances
- Moments
- Binomial Distribution
- Poisson Distribution
- Hypergeometric Distribution
- Negative Binomial Distribution
- Geometric Distribution
- Multinomial Distribution
- Continuous Probability Distributions
- Probability Density Functions (PDFs)
- Cumulative Distribution Functions (CDFs)
- Normal Distribution
- Exponential Distribution
- Central Limit Theorem
- Beta Distribution
- Gamma Distribution
- Chi-Square Distribution
- Uniform Distribution
- Log-Normal Distribution
- Weibull Distribution
- Advanced Probability Topics
- Joint and Marginal Distributions
- Bivariate Distributions
- Independence
- Correlation
- Functions of Random Variables
- Transformation Techniques
- Moment-Generating Functions
- Order Statistics
- Reliability
- Life Data Analysis
- Convergence in Probability
- Almost Sure Convergence
- Law of Large Numbers
- Statistician
- Data Scientist
- Uncertainty
- Confidence
- Expertise
- Knowledge

In the context of the course on Foundations of Probability, Discrete and Continuous Probability Distributions, and Advanced Topics in Probability, let's explore a use case related to analyzing risk using probability distributions. This use case involves the application of probability concepts to assess and manage risk in financial investments.

### Description:

In this use case, we will focus on modeling the risk associated with a financial investment using probability distributions. We will consider a hypothetical investment in a stock and assess the potential returns and risks associated with it by modeling the stock's price movement.

### Key Components:

1. Foundations of Probability: Understanding the basic concepts of probability, such as sample space, events, and probability axioms, is essential for modeling and assessing risk.
2. Discrete and Continuous Probability Distributions: Knowledge of discrete and continuous probability distributions, including their probability mass functions (PMFs) and probability density functions (PDFs), is crucial for modeling financial data.
3. Expected Value and Variance: Calculating the expected value and variance of a random variable helps assess the average return and risk associated with an investment.
4. Advanced Topics in Probability: Concepts such as joint and marginal distributions, functions of random variables, and convergence concepts are useful for analyzing the relationship between multiple financial instruments.

### Python Code Example (Analyzing Investment Risk):

```
1 import numpy as np
2 import matplotlib.pyplot as plt
3 from scipy.stats import norm
4
5 # Define parameters of the stock's return distribution
6 mean_return = 0.08 # Mean return
7 volatility = 0.15 # Annual volatility (standard deviation)
8
9 # Generate a range of possible annual returns
10 returns = np.linspace(-0.3, 0.3, 1000)
11
12 # Calculate the probability density function (PDF) using the normal distribution
13 pdf_values = norm.pdf(returns, loc=mean_return, scale=volatility)
14
15 # Plot the PDF of the stock's returns
16 plt.figure(figsize=(10, 6))
17 plt.plot(returns, pdf_values, label='PDF', color='blue')
18 plt.xlabel('Annual Return')
19 plt.ylabel('Probability Density')
20 plt.title('Probability Density Function (PDF) of Stock Returns')
21 plt.legend()
22 plt.grid(True)
23
24 # Calculate and print the expected return and risk (standard deviation)
25 expected_return = mean_return
26 risk = volatility
27 print(f'Expected Return: {expected_return:.2%}')
28 print(f'Risk (Standard Deviation): {risk:.2%}')
29
30 plt.show()
```

In this code, we model the probability distribution of a stock's annual returns using a normal distribution with a specified mean return and volatility. We calculate and display the probability density function (PDF) of returns and then calculate the expected return and risk (standard deviation) as measures of investment risk.

This use case demonstrates how probability concepts can be applied to assess investment risk, allowing investors and analysts to make informed decisions based on the expected return and risk associated with a financial instrument.

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## TEXTE DE DESCRIPTION DU COURS

Welcome to "Probability Unveiled: Exploring the Foundations and Beyond." This course is your gateway to unraveling the fascinating world of probability theory from its historical origins to its advanced applications in modern statistics and data science. Prepare to embark on a captivating journey through the fundamental principles, discrete and continuous probability distributions, and advanced probability concepts.

As your voyage begins, we'll take you on a historical and philosophical overview of probability theory. You'll discover the intriguing origins of probability theory and explore the timeless debate between frequentist and Bayesian interpretations. These philosophical underpinnings will provide you with a profound perspective on probability.

In the section on basic concepts and definitions, you'll delve into the core building blocks of probability theory. Learn to conceptualize experiments, sample spaces, and events, and gain a solid grasp of the probability axioms that form the bedrock of probability theory. Rules of probability, including the addition and multiplication rules, as well as conditional probability and independence, will become your second nature.

Counting methods will equip you with powerful tools for probability analysis. Understand permutations and combinations and witness their real-world applications in solving intricate probability problems.

The course then transitions into discrete probability distributions, where you'll explore the basics of probability mass functions (PMFs) and delve into expected values, variances, and moments. Dive deep into the binomial and Poisson distributions, examining their assumptions, properties, and practical applications. Discover the hypergeometric and negative binomial distributions and learn how to wield them effectively in various scenarios. Gain insights into other discrete distributions such as the geometric and multinomial distributions, and understand their significance.

Continuing your journey, you'll explore continuous probability distributions. Get comfortable with probability density functions (PDFs) and cumulative distribution functions (CDFs), and master the calculations of expected values, variances, and moments for continuous distributions. The normal and exponential distributions will take center stage, revealing their properties and significance, as well as their role in the Central Limit Theorem. Delve into the beta, gamma, and chi-square distributions, unraveling their relationships and their applications in statistical inference. Explore other continuous distributions like the uniform, log-normal, and Weibull distributions, gaining insights into their contexts and relevance.

The final part of the course introduces advanced topics in probability. Understand the intricacies of joint and marginal distributions, exploring bivariate distributions, independence, and correlation. Master functions of random variables, including transformation techniques and moment-generating functions. Explore the distribution of order statistics and its applications in reliability and life data analysis. Finally, grasp the concept of convergence in probability and almost sure convergence, and discover the profound implications of the law of large numbers.

Whether you're a budding statistician, a data scientist, or simply someone intrigued by the art of uncertainty, this course will equip you with the knowledge and tools to navigate the intricate world of probability with confidence and expertise.

# DESIGN DE LA CARD

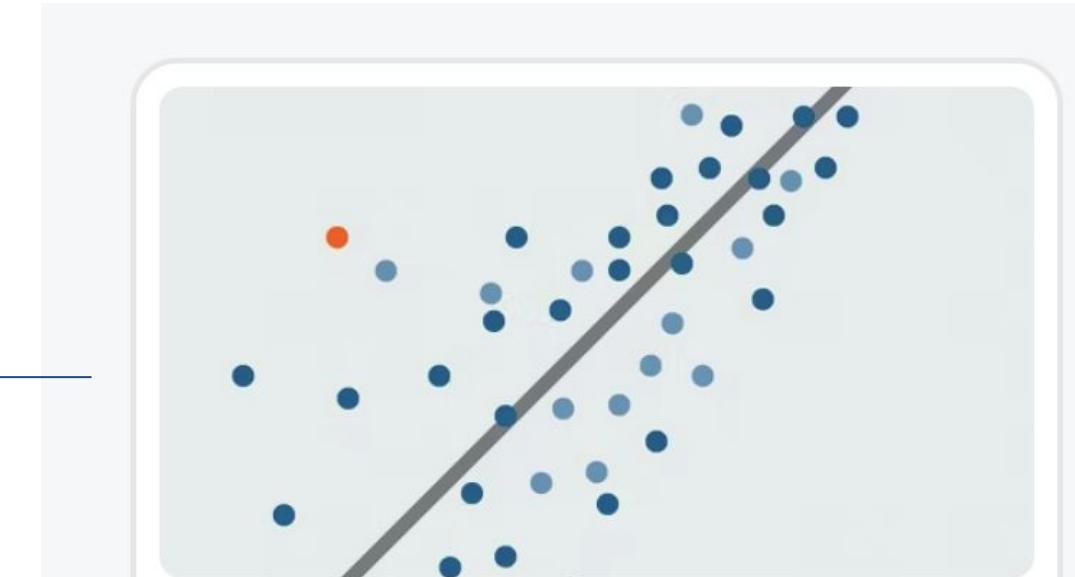
$$\frac{\partial}{\partial \theta} \ln f_{a,\sigma^2}(\xi_1) = \frac{(\xi_1 - a)}{\sigma^2} f_{a,\sigma^2}(\xi_1) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(\xi_1 - a)^2}{2\sigma^2}\right)$$
$$\int_{\mathbb{R}_n} T(x) \cdot \frac{\partial}{\partial \theta} f(x, \theta) dx = M\left(T(\xi) \cdot \frac{\partial}{\partial \theta} \ln L(\xi, \theta)\right) \int_{\mathbb{R}_n} T(x) f(x, \theta) dx$$
$$\int_{\mathbb{R}_n} T(x) \cdot \left( \frac{\partial}{\partial \theta} \ln L(x, \theta) \right) \cdot f(x, \theta) dx = \int_{\mathbb{R}_n} T(x) \left( \frac{\frac{\partial}{\partial \theta} f(x, \theta)}{f(x, \theta)} \right) f(x, \theta) dx$$
$$\frac{\partial}{\partial \theta} \ln T(\xi) = \frac{\partial}{\partial \theta} \int_{\mathbb{R}_n} T(x) f(x, \theta) dx = \int_{\mathbb{R}_n} \frac{\partial}{\partial \theta} T(x) f(x, \theta) dx$$



Author: Baptiste Mokas, Weeki

Course Name: Simple Linear Regression

#ProbabilityFoundations  
#ProbabilityDistributions  
#AdvancedProbability



Duke University

Linear Regression and Modeling

Compétences que vous acquerez: Probability & Statistics, Regression, Business Analysis, Data Analysis, General Statistics, Statistical Analysis,...

★ 4.8 (1.7k avis)

Débutant · Course · 1 à 4 semaines