LES FACULTÉS
DE L'UNIVERSITÉ
CATHOLIQUE DE LILLE

Stochastic dynamics & probability

MEASURE THEORY

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KNOWLEDGE TREE

Part 1: Foundations of Real Analysis and Measure Theory

1.1 Preliminary Notions

- Real Numbers and Their Properties
- Sequences and Series
- The Structure of R

1.2 Principles of Set Theory

- Axioms of Set Theory
- Relations, Functions, and Operations
- The Power Set and Infinite Sets
- Definitions of Sets and Elements
- Set Operations (Union, Intersection, Complement)
- Cardinality and Countable Sets

1.3 Measure Theory Basics

- Motivation and Introduction
- Sigma-Algebras
- Lebesgue Measure
- Construction of Product Measures
- Measurable Sets with Respect to Lebesgue Measure
- Definition of Sigma-Algebra
- Measure and Measure Spaces
- Measurable Sets and Functions

Part 2: Integral Transformations, Function Spaces, and Lebesgue Integration

2.1 Basics of Integration

- Riemann Integration
- Lebesgue Integration
- Construction of Lebesgue Measure
- Properties of Lebesgue Measure
- Definition of Lebesgue Integral
- Properties of Lebesgue Integrals
- Convergence Theorems

2.2 Fourier Analysis

- Fourier Series
- Fourier Transform
- Applications and Extensions

2.3 Function Spaces and Lp Norms

- Banach and Hilbert Spaces
- Lp Spaces
- Duality and Functionals
- Convergence in Measure
- Convergence Almost Everywhere
- Egorov's Theorem

Part 3: Advanced Topics in Measure Theory and Probability Spaces

3.1 Product Measures, Differentiation, and Probability Spaces

- Product Measures and Fubini's Theorem
- Applications in Probability and Analysis
- The Fundamental Theorem of Calculus
- BV Functions
- The Lebesgue-Radon-Nikodym theorem
- Probability Measure Definition
- Probability Spaces and Events

3.2 Signed Measures, Decompositions, and Functions of Bounded Variation

- Hahn Decomposition
- Radon-Nikodym Derivative
- Complex Measures
- Absolute Continuity and Singularity

Part 4: Analytical Techniques, Applications, and Advanced Model Selection

4.1 Topology, Metric Spaces, and Scatterplots

- Basic Concepts of Topology
- Compactness and Continuity
- Baire's Category Theorem
- Absolute Continuity and Singularity
- Radon-Nikodym Theorem Statement
- Applications in Probability Theory

4.2 Applications in Probability and Measure-Theoretic Probability

- Random Variables and Measure Theory
- Expectation and Integration
- The Law of Large Numbers and Central Limit Theorem
- Probability Spaces with Measure Theory
- Laws of Large Numbers
- Central Limit Theorem

4.3 Special Topics, Integration Formulas, and Further Studies

- Analytical Number Theory
- Ergodic Theory
- Potential Theory
- Convergence Theorems
- Integrals Depending on a Parameter
- Multiple Integrals
- L^p Spaces
- Fourier Inversion Formula
- Formula Sheet with Key Concepts

OUR STORY LINE

Probability and Statistics

STEP -1 _ PROGRAM INTRODUCTION STEP 0 _ FOUNDATIONS STEP 1 _ THEORY OF SYSTEMS	STEP 2 _ STOCHASTIC DYNAMICS & PROBABILITY STEP 3 _ DATA OBSERVATION	STEP 4 _ INFERENCE & ESTIMATION THEORY	STEP 5 _ LINEAR MODEL EXAMPLES	STEP 6 _ OTHER MODEL EXAMPLES	STEP 7 _ NON LINEAR MODELS

OUR STORY LINE

Probability and Statistics

STEP -1 _ PROGRAM INTRODUCTION
- -1 - PROGRAM INTRODUCTION

STEP 0 _ FOUNDATIONS
- 0.1 - ELEMENTS OF CALCULUS & TOOLS
- 0.2 - EPISTEMOLOGY & THEORY OF KNOWLEDGE

STEP 1 _ THEORY OF SYSTEMS
- 1.1 - DYNAMICAL SYSTEMS
- 1.2 - COMPLEX ADAPTIVE SYSTEMS

STEP 2 _ STOCHASTIC DYNAMICS & PROBABILITY
- 2.1 - MEASURE THEORY
- 2.2 - PROBABILITY THEORY
- 2.3 - USUAL PROBABILITY DISTRIBUTIONS
- 2.4 - ASYMPTOTIC STATISTICS
- 2.5 - STOCHASTIC PROCESS & TIME SERIES
- 2.6 - INFORMATION GEOMETRY

STEP 3 _ DATA OBSERVATION
- 3.1 - DESCRIPTIVE STATISTICS & DATAVIZUALISATION
- 3.2 - EXPLORATORY DATA ANALYSIS

STEP 4 _ INFERENCE & ESTIMATION THEORY
- 4.1 - PARAMETERS ESTIMATIONS & LEARNING
- 4.2 - EXPERIMENTAL DESIGN & HYPOTHESIS TESTING
- 4.4 - DECISION TREES & MODEL SELECTION
- 4.5 - BAYESIAN INFERENCES

STEP 5 _ LINEAR MODEL EXAMPLES
- 5.1 - SIMPLE LINEAR REGRESSION
- 5.2 - MULTIPLE LINEAR REGRESSION
- 5.3 - OTHER REGRESSIONS MODELS

STEP 6 _ OTHER MODEL EXAMPLES
- 6.1 - USUAL UNIVARIATE TESTING
- 6.2 - USUAL MULTIVARIATE TESTING
- 6.3 - NON PARAMETRIC STATISTICS

STEP 7 _ NON LINEAR MODELS
- 7.1 - PROBABILISTIC GRAPHICAL MODELS
- 7.2 - PERCOLATION THEORY
- 7.3 - SPATIAL STATISTICS
- 7.4 - EXTREM VALUE THEORY
- 7.5 - AGENT BASED MODELING
- 7.6 - NETWORK DYNAMICS

KEYWORDS (NEW)

Espace L^p
Théorème de Radon-Nikodym
Théorème de convergence presque partout
Théorème de convergence uniforme
Théorème de Fubini
Tribu
Mesure spectrale
Théorie des ensembles
Théorème de convergence monotone
Théorème de l'égalité de Jensen
Théorème de Green
Paramètre
Mesure invariante
Théorème de Carathéodory
Convergence en moyenne quadratique
Théorème de Riesz-Markov-Kakutani
Théorème de Jensen
Mesure
Mesure extérieure
Théorème de Cameron-Martin
Théorème de convergence faible
Théorème de Lebesgue
Intégrabilité
Intégrale positive
Fonction mesurable
Mesure de Hausdorff
Théorème de Borel-Cantelli
Théorème de convergence dominée
Théorème de Vitali
Intégrale signée
Intégrale de Henstock-Kurzweil
Théorème de Bernstein
Mesure de Borel
Mesure absolument continue
Théorème de Bolzano-Weierstrass
Théorie de la mesure
Intégrale simple
Intégrale de Lebesgue impropre
Mesure sigma-finie
Espace euclidien
Fonction mesurable négative
Mesure extérieure de Carathéodory
Théorème de Cantor
Mesure atomique
Intégrale de Lebesgue-Stieltjes
Mesure de probabilité
Mesure de Jordan
Fonction mesurable positive
Fonction mesurable essentielle
Théorème de Blackwell
Mesure de Borel
Dérivée radon-nikodym
Mesure de Lebesgue
Mesure de capacité
Espace mesurable
Intégrale de Choquet
Espace de Banach
Ensemble mesurable
Intégrale négative
Intégrale de Pettis
Intégrale de Stieltjes
Fonction caractéristique
Intégrale de Daniell
Espace de Hilbert
Ensemble négligeable
Intégrale de Young
Espace de mesure
Espace des réels
Intégrale de Lebesgue
Fonction de mesure

Real Numbers and Their Properties

Part 1: Foundations of Real Analysis and Measure Theory

1.1 Preliminary Notions

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The Structure of R

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Axioms of Set Theory

Part 1: Foundations of Real Analysis and Measure Theory

1.2 Principles of Set Theory

Relations, Functions, and Operations

Part 1: Foundations of Real Analysis and Measure Theory

1.2 Principles of Set Theory

The Power Set and Infinite Sets

Part 1: Foundations of Real Analysis and Measure Theory

1.2 Principles of Set Theory

Definitions of Sets and Elements

Part 1: Foundations of Real Analysis and Measure Theory

1.2 Principles of Set Theory

Set Operations (Union, Intersection, Complement)

Part 1: Foundations of Real Analysis and Measure Theory

1.2 Principles of Set Theory

Cardinality and Countable Sets

Part 1: Foundations of Real Analysis and Measure Theory

1.2 Principles of Set Theory

Motivation and Introduction

Part 1: Foundations of Real Analysis and Measure Theory

1.3 Measure Theory Basics

Sigma-Algebras

Part 1: Foundations of Real Analysis and Measure Theory

1.3 Measure Theory Basics

Lebesgue Measure

Part 1: Foundations of Real Analysis and Measure Theory

1.3 Measure Theory Basics

Construction of Product Measures

Part 1: Foundations of Real Analysis and Measure Theory

1.3 Measure Theory Basics

Measurable Sets with Respect to Lebesgue Measure

Part 1: Foundations of Real Analysis and Measure Theory

1.3 Measure Theory Basics

Definition of Sigma-Algebra

Part 1: Foundations of Real Analysis and Measure Theory

1.3 Measure Theory Basics

Part 1: Foundations of Real Analysis and Measure Theory

1.3 Measure Theory Basics

Measurable Sets and Functions

Part 1: Foundations of Real Analysis and Measure Theory

1.3 Measure Theory Basics

Riemann Integration

Part 2: Integral Transformations, Function Spaces, and Lebesgue Integration

2.1 Basics of Integration

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2.1 Basics of Integration

Properties of Lebesgue Measure

Part 2: Integral Transformations, Function Spaces, and Lebesgue Integration

2.1 Basics of Integration

Definition of Lebesgue Integral

Part 2: Integral Transformations, Function Spaces, and Lebesgue Integration

2.1 Basics of Integration

Properties of Lebesgue Integrals

Part 2: Integral Transformations, Function Spaces, and Lebesgue Integration

2.1 Basics of Integration

Convergence Theorems

Part 2: Integral Transformations, Function Spaces, and Lebesgue Integration

2.1 Basics of Integration

Fourier Series

Part 2: Integral Transformations, Function Spaces, and Lebesgue Integration

2.2 Fourier Analysis

Fourier Transform

Part 2: Integral Transformations, Function Spaces, and Lebesgue Integration

2.2 Fourier Analysis

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Part 2: Integral Transformations, Function Spaces, and Lebesgue Integration

2.3 Function Spaces and Lp Norms

Lp Spaces

Part 2: Integral Transformations, Function Spaces, and Lebesgue Integration

2.3 Function Spaces and Lp Norms

Duality and Functionals

Part 2: Integral Transformations, Function Spaces, and Lebesgue Integration

2.3 Function Spaces and Lp Norms

Convergence in Measure

Part 2: Integral Transformations, Function Spaces, and Lebesgue Integration

2.3 Function Spaces and Lp Norms

Convergence Almost Everywhere

Part 2: Integral Transformations, Function Spaces, and Lebesgue Integration

2.3 Function Spaces and Lp Norms

Egorov's Theorem

Part 2: Integral Transformations, Function Spaces, and Lebesgue Integration

2.3 Function Spaces and Lp Norms

Product Measures and Fubini's Theorem

Part 3: Advanced Topics in Measure Theory and Probability Spaces

3.1 Product Measures, Differentiation, and Probability Spaces

Part 3: Advanced Topics in Measure Theory and Probability Spaces

3.1 Product Measures, Differentiation, and Probability Spaces

The Fundamental Theorem of Calculus

Part 3: Advanced Topics in Measure Theory and Probability Spaces

3.1 Product Measures, Differentiation, and Probability Spaces

BV Functions

Part 3: Advanced Topics in Measure Theory and Probability Spaces

3.1 Product Measures, Differentiation, and Probability Spaces

The Lebesgue-Radon-Nikodym theorem

Part 3: Advanced Topics in Measure Theory and Probability Spaces

3.1 Product Measures, Differentiation, and Probability Spaces

Probability Measure Definition

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3.1 Product Measures, Differentiation, and Probability Spaces

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Hahn Decomposition

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3.2 Signed Measures, Decompositions, and Functions of Bounded Variation

Radon-Nikodym Derivative

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3.2 Signed Measures, Decompositions, and Functions of Bounded Variation

Complex Measures

Part 3: Advanced Topics in Measure Theory and Probability Spaces

3.2 Signed Measures, Decompositions, and Functions of Bounded Variation

Absolute Continuity and Singularity

Part 3: Advanced Topics in Measure Theory and Probability Spaces

3.2 Signed Measures, Decompositions, and Functions of Bounded Variation

Basic Concepts of Topology

Part 4: Analytical Techniques, Applications, and Advanced Model Selection

4.1 Topology, Metric Spaces, and Scatterplots

Compactness and Continuity

Part 4: Analytical Techniques, Applications, and Advanced Model Selection

4.1 Topology, Metric Spaces, and Scatterplots

Baire's Category Theorem

Part 4: Analytical Techniques, Applications, and Advanced Model Selection

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Radon-Nikodym Theorem Statement

Part 4: Analytical Techniques, Applications, and Advanced Model Selection

4.1 Topology, Metric Spaces, and Scatterplots

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Part 4: Analytical Techniques, Applications, and Advanced Model Selection

4.2 Applications in Probability and Measure-Theoretic Probability

Part 4: Analytical Techniques, Applications, and Advanced Model Selection

4.2 Applications in Probability and Measure-Theoretic Probability

The Law of Large Numbers and Central Limit Theorem

Part 4: Analytical Techniques, Applications, and Advanced Model Selection

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4.2 Applications in Probability and Measure-Theoretic Probability

Central Limit Theorem

Part 4: Analytical Techniques, Applications, and Advanced Model Selection

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4.2 Applications in Probability and Measure-Theoretic Probability

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4.3 Special Topics, Integration Formulas, and Further Studies

Part 4: Analytical Techniques, Applications, and Advanced Model Selection

4.3 Special Topics, Integration Formulas, and Further Studies

Convergence Theorems

Part 4: Analytical Techniques, Applications, and Advanced Model Selection

4.3 Special Topics, Integration Formulas, and Further Studies

Integrals Depending on a Parameter

Part 4: Analytical Techniques, Applications, and Advanced Model Selection

4.3 Special Topics, Integration Formulas, and Further Studies

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L^p Spaces

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Fourier Inversion Formula

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4.3 Special Topics, Integration Formulas, and Further Studies

Use Case: Probability Distribution Modeling Using Measure Theory

In the context of the course on Measure Theory, Lebesgue Measure and Integration, Probability Measures and Distributions, and Advanced Topics, let's explore a use case related to modeling probability distributions using measure theory. This use case involves the application of measure theory concepts in understanding and modeling probability distributions.

Description:

In this use case, we will focus on modeling a probability distribution, specifically a continuous probability distribution, using the tools and concepts of measure theory. We will use the Lebesgue integral to calculate probabilities and expected values, and explore the properties of probability distributions.

Key Components:

1. Measure Theory Fundamentals: Understanding the basics of sets, sigma-algebras, measures, and measurable sets is essential for modeling probability distributions using measure theory.

2. Lebesgue Measure and Integration: Lebesgue measure and the Lebesgue integral are fundamental concepts for modeling continuous probability distributions and calculating probabilities.

3. Probability Distributions: Understanding discrete and continuous probability distributions, probability density functions (PDFs), and cumulative distribution functions (CDFs) is crucial for modeling real-world phenomena.

4. Expected Value and Variance: Calculating expected values and variances of random variables is an important part of probability distribution modeling.

Python Code Example (Modeling a Continuous Probability Distribution):

```python
import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import norm

# Define parameters of the normal distribution
mu = 0  # Mean
sigma = 1  # Standard deviation

# Generate data points and PDF values for the normal distribution
x = np.linspace(-5, 5, 1000)
pdf_values = norm.pdf(x, loc=mu, scale=sigma)

# Plot the probability density function (PDF) of the normal distribution
plt.figure(figsize=(10, 6))
plt.plot(x, pdf_values, label='Normal PDF', color='blue')
plt.xlabel('Random Variable (x)')
plt.ylabel('Probability Density')
plt.title('Probability Density Function (PDF) of the Normal Distribution')
plt.legend()
plt.grid(True)

# Calculate and print the expected value and variance
expected_value = norm.expect(loc=mu, scale=sigma)
variance = norm.var(loc=mu, scale=sigma)
print(f'Expected Value: {expected_value}')
print(f'Variance: {variance}')

plt.show()
```

In this code, we use the probability density function (PDF) of the normal distribution to model a continuous probability distribution. We calculate and display the PDF, and then calculate the expected value and variance, which are fundamental properties of probability distributions.

This use case demonstrates how measure theory concepts can be applied to model and analyze probability distributions, providing a rigorous foundation for understanding and working with real-world data and phenomena.

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TEXTE DE DESCRIPTION DU COURS

Welcome to the enlightening journey through "Measure Theory and Probability: Unveiling the Mathematical Foundations." This course invites you to immerse yourself in the profound world of measure theory, probability theory, and their wide-ranging applications. Prepare to embark on an intellectual odyssey that will equip you with the tools to comprehend and navigate the intricacies of probability, randomness, and uncertainty.

As you embark on this course, your voyage begins with a deep dive into set theory fundamentals. You will explore the essence of sets, elements, set operations, and the concept of cardinality. The construction of real numbers and the notion of convergence and limits of sequences will introduce you to the fundamental building blocks of analysis. Completeness of real numbers will emerge as a pivotal concept, bridging the abstract with the concrete.

Sigma-algebras and measure spaces will become your compass as you enter the realm of measure theory. You'll gain a comprehensive understanding of sigma-algebras, measures, and measurable sets and functions, paving the way for precise and rigorous mathematical reasoning.

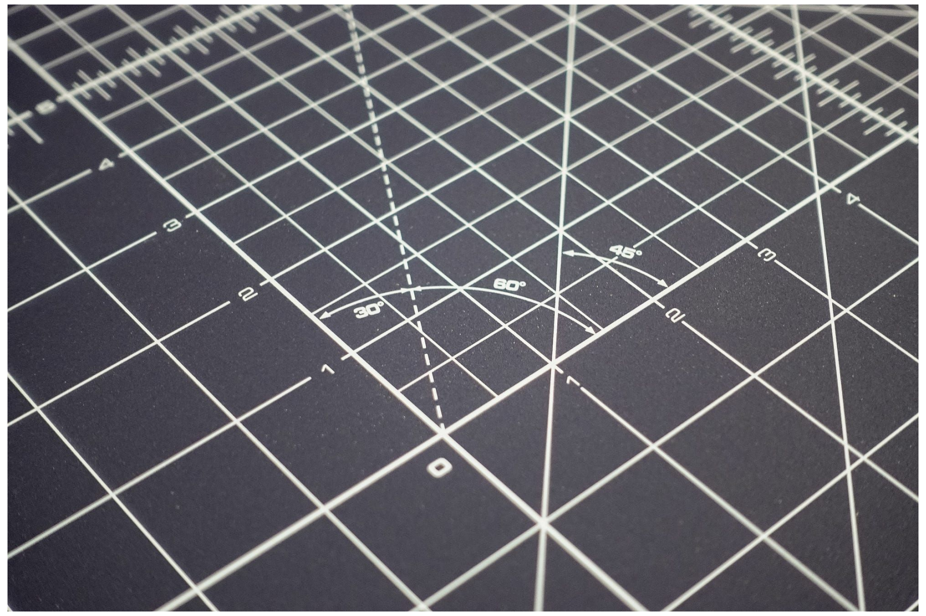
The second part of the course delves into Lebesgue measure and integration, shedding light on the construction and properties of this foundational measure. You'll master the definition of Lebesgue integrals, their properties, and the powerful Lebesgue Dominated Convergence Theorem. Convergence in measure will become second nature, alongside concepts such as convergence almost everywhere and Egorov's Theorem.

The course then transitions seamlessly into probability measures and distributions. Probability spaces will be your foundation, with a clear definition of probability measures and their axioms. You'll delve into probability distributions, distinguishing between discrete and continuous distributions, and gaining insights into probability density functions (PDFs) and cumulative distribution functions (CDFs). Expected value and variance will provide the mathematical tools needed for probabilistic analysis.

In the final part of the course, you'll venture into advanced topics, including the Radon-Nikodym Theorem, which explores absolute continuity and singularity. The measure-theoretic foundation of probability theory will unfold, connecting probability spaces with measure theory. You'll explore the laws of large numbers and the central limit theorem, uncovering the mathematical underpinnings of statistical inference. Stochastic processes will complete your journey, with an introduction to Markov chains, martingales, and the fascinating world of Brownian motion.

Whether you're an aspiring mathematician, a data scientist, a statistician, or simply someone intrigued by the elegant dance between measure theory and probability, this course offers a comprehensive exploration of the mathematical foundations that underpin the world of uncertainty and randomness.

DESIGN DE LA CARD



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Course Name: Simple Linear Regression

#MeasureTheory
#ProbabilityFoundations
#StochasticProcesses