LES FACULTÉS
DE L'UNIVERSITÉ
CATHOLIQUE DE LILLE

Models examples

STOCHASTIC PROCESSES & TIME SERIES

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KNOWLEDGE TREE

Part 1: Fundamental Discrete-Time Processes

1.1 Random Walks and Drifts

- Simple random walk and properties
- Random walks with drift: definition and real-world applications

1.2 Markov Chains: Basics

- Transition matrix and state space
- Chapman-Kolmogorov equations and n-step transition probabilities

1.3 Markov Chains: Classification of States

- Transient and recurrent states
- Periodic and aperiodic states; ergodicity

1.4 Markov Chains: Long-Run Behavior

- Stationary distributions
- Convergence to equilibrium

Part 2: Fundamental Continuous-Time Processes

2.1 Poisson Processes

- Homogeneous Poisson process
- Non-homogeneous and compound Poisson processes

2.2 Birth-Death Processes

- Introduction and dynamics
- Applications in population studies and queuing systems

2.3 Brownian Motion and Wiener Process

- Basic properties and sample paths
- Quadratic variation and reflection principle

2.4 Exponential and Gamma Processes

- Memorylessness property
- Applications in reliability and queuing

Part 3: Advanced Discrete-Time Processes

3.1 Branching Processes

- Galton-Watson process and extinction probabilities
- Applications in population dynamics

3.2 Renewal Processes

- Introduction and renewal function
- Key renewal theorem

3.3 Hidden Markov Models

- Structure and components
- Algorithms: Forward-Backward, Viterbi

3.4 Time Series and Autoregressive Models

- AR, MA, and ARMA models
- Stationarity and invertibility

4.2 Independence, Martingales, and Conditional Expectations

- Independence of Events and Random Variables
- Borel-Cantelli Lemma
- Law of a Tuple of Independent Variables
- Concept of Martingale
- Convergence of Martingales
- Series of Independent Random Variables
- Convergence of Conditional Expectations
- Stopping Time
- Illustration with a Random Walk

Part 4: Advanced Continuous-Time Processes

4.1 Jump Processes and Lévy Processes

- Compound Poisson and generalized processes
- Stability and infinite divisibility

4.2 Martingales in Continuous Time

- Martingale properties and stopping times
- Doob's maximal inequality

4.3 Diffusion Processes

- Introduction and Ito's lemma
- Stochastic differential equations and applications

4.4 Queueing Processes

- M/M/1 and its extensions
- Traffic intensity, average queue length, and waiting time

KEYWORDS (NEW)

Classification des états | Processus stochastique | Processus de Wiener (Brownien) | Calcul stochastique | Densité de probabilité | Propriété de Markov

Méthode de Monte Carlo | Théorème de Galton-Watson | Processus markoviens de saut | Chaîne de Markov | Processus de Poisson | Théorème de Feller

Processus de Bernoulli | Théorème de Martingale | Théorème de Markov | Théorème de Wald | Théorème de Slutsky | Processus gaussien

Processus de renouvellement | Théorème de Wald-Wolfowitz

KEYWORDS

- Stochastic Processes
- Discrete-Time Processes
- Continuous-Time Processes
- Properties
- Classifications
- Real-World Applications
- Random Walks
- Drifts
- Simple Random Walks
- Random Walks with Drift
- Markov Chains
- Transition Matrices
- State Spaces
- Chapman-Kolmogorov Equations
- Classification of States
- Transient States
- Recurrent States
- Periodicity
- Ergodicity
- Stationary Distributions
- Convergence to Equilibrium
- Poisson Processes
- Homogeneous
- Non-Homogeneous
- Compound Poisson Processes
- Birth-Death Processes
- Population Studies
- Queuing Systems
- Brownian Motion
- Wiener Processes
- Sample Paths
- Quadratic Variation
- Exponential Processes
- Gamma Processes
- Memorylessness
- Reliability
- Advanced Discrete-Time Processes
- Branching Processes
- Galton-Watson Process
- Extinction Probabilities
- Renewal Processes
- Renewal Function
- Renewal Theorem
- Hidden Markov Models
- Forward-Backward Algorithm
- Viterbi Algorithm
- Time Series
- Autoregressive Models
- AR
- MA
- ARMA Models
- Stationarity
- Invertibility
- Advanced Continuous-Time Processes
- Jump Processes
- Lévy Processes
- Generalized Processes
- Infinite Divisibility
- Martingales
- Doob's Maximal Inequality
- Diffusion Processes
- Ito's Lemma
- Stochastic Differential Equations
- Finance
- Engineering
- Biology
- Challenges
- Real-World
- Knowledge
- Skills

Use Case with python

Use Case: Modeling Queuing Systems with M/M/1 Queue

In the context of the course on Fundamental Discrete-Time Processes, Fundamental Continuous-Time Processes, Advanced Discrete-Time Processes, and Advanced Continuous-Time Processes, let's explore a use case related to modeling queuing systems using the M/M/1 queueing model. This use case involves the application of stochastic processes to analyze and optimize the performance of a queuing system.

Description:

In this use case, we will focus on modeling a simple queuing system using the M/M/1 queueing model, which is a widely used model for analyzing the behavior of single-server queues. We will explore how stochastic processes can help us understand key performance metrics such as average queue length and waiting time.

Key Components:

Fundamental Discrete-Time Processes: Understanding concepts related to random walks, Markov chains, and their properties is essential for modeling queuing systems.

Fundamental Continuous-Time Processes: Knowledge of Poisson processes and Brownian motion is valuable for understanding the arrival and service processes in queuing systems.

Advanced Discrete-Time Processes: Concepts related to renewal processes and branching processes may be relevant for modeling complex queuing scenarios.

Advanced Continuous-Time Processes: Understanding jump processes, martingales, and diffusion processes can provide insights into more complex queuing models.

Python Code Example (M/M/1 Queue Simulation):

```python
import numpy as np
import matplotlib.pyplot as plt

# Parameters of the M/M/1 queue
arrival_rate = 2.0  # Average arrival rate (lambda)
service_rate = 3.0  # Average service rate (mu)
rho = arrival_rate / service_rate  # Traffic intensity

# Simulation parameters
sim_time = 1000  # Total simulation time
time_step = 0.01  # Time step for simulation
num_customers = int(sim_time / time_step)  # Number of customers

# Initialize variables
arrival_times = np.zeros(num_customers)
service_times = np.zeros(num_customers)
waiting_times = np.zeros(num_customers)

# Simulate customer arrivals and service times
for i in range(1, num_customers):
    arrival_times[i] = arrival_times[i - 1] + np.random.exponential(1 / arrival_rate)
    service_times[i] = np.random.exponential(1 / service_rate)
    waiting_times[i] = max(0, waiting_times[i - 1] + service_times[i - 1] - arrival_times[i]) + service_times[i]

# Calculate average queue length and waiting time
average_queue_length = np.mean(waiting_times / service_times)
average_waiting_time = np.mean(waiting_times)

print(f'Average Queue Length: {average_queue_length:.2f}')
print(f'Average Waiting Time: {average_waiting_time:.2f} time units')

# Plot the queue length over time
plt.figure(figsize=(10, 6))
plt.plot(np.arange(0, sim_time, time_step), waiting_times)
plt.xlabel('Time')
plt.ylabel('Queue Length')
plt.title('M/M/1 Queue Length Over Time')
plt.grid(True)
plt.show()
```

In this code, we simulate an M/M/1 queueing system, where customers arrive following a Poisson process and are served by a single server. We calculate performance metrics such as the average queue length and waiting time to analyze the behavior of the queuing system.

This use case demonstrates how stochastic processes can be applied to model and analyze queuing systems, helping businesses optimize resource allocation and customer service.

TOP 20 REFERENCES

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TEXTE DE DESCRIPTION DU COURS

Welcome to "Stochastic Processes Unveiled: From Fundamentals to Advanced Applications." This course is your gateway to exploring the captivating world of stochastic processes, bridging the gap between theory and real-world applications. Dive into the realms of discrete-time and continuous-time processes, gaining insights into their properties, classifications, and practical significance.

Your journey commences with "Fundamental Discrete-Time Processes." Delve into the intriguing realm of random walks and drifts, unraveling the dynamics of simple random walks and their essential properties. Venture into the world of random walks with drift, understanding their definition and real-world applications. The course progresses to "Markov Chains: Basics," where you'll become proficient in transition matrices, state spaces, and Chapman-Kolmogorov equations. Explore the classification of states, distinguishing between transient and recurrent states, and dive into the concepts of periodicity and ergodicity. Finally, uncover the long-run behavior of Markov chains, including stationary distributions and convergence to equilibrium.

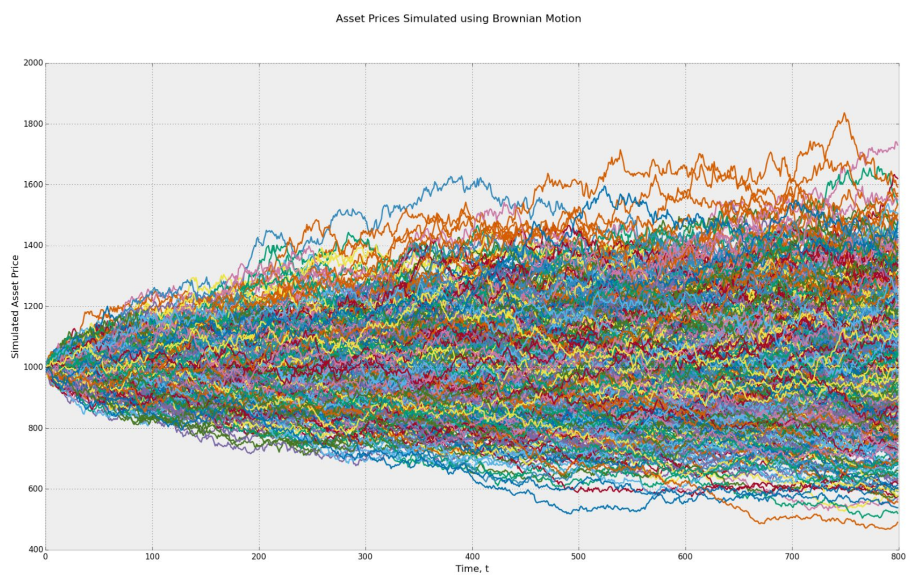
Transitioning to "Fundamental Continuous-Time Processes," you'll embark on a journey into the continuous-time realm. Master the intricacies of Poisson processes, examining both homogeneous and non-homogeneous variants, as well as compound Poisson processes. Discover birth-death processes, their dynamics, and their wide-ranging applications in population studies and queuing systems. The exploration continues with an in-depth understanding of Brownian motion and Wiener processes, exploring their fundamental properties, sample paths, and the quadratic variation. Finally, delve into exponential and gamma processes, grasping their memorylessness property and their vital role in reliability and queuing.

As your expertise grows, the course delves into "Advanced Discrete-Time Processes." Explore branching processes, including the Galton-Watson process and extinction probabilities, and witness their applications in the dynamic world of population dynamics. Uncover the intricacies of renewal processes, mastering the renewal function and the key renewal theorem. Finally, venture into the world of hidden Markov models, dissecting their structure and components, and mastering algorithms like the Forward-Backward and Viterbi algorithms. The course culminates with a deep dive into time series and autoregressive models, including AR, MA, and ARMA models, and delves into stationarity and invertibility.

The final part of the course, "Advanced Continuous-Time Processes," takes you on a journey through jump processes and Lévy processes, including compound Poisson and generalized processes. Explore their stability and the fascinating concept of infinite divisibility. Dive into martingales in continuous time, mastering martingale properties, and stopping times, all while applying Doob's maximal inequality. Finally, grasp the intricacies of diffusion processes, including Ito's lemma and stochastic differential equations, and their wide-ranging applications.

Whether you're a mathematician, a data scientist, or a professional in fields like finance, engineering, or biology, this course will equip you with the knowledge and skills to master stochastic processes and apply them to a diverse array of real-world challenges.

DESIGN DE LA CARD



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Course Name: Simple Linear Regression

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#ContinuousTimeProcesses