LES FACULTÉS
DE L'UNIVERSITÉ
CATHOLIQUE DE LILLE

Non linear models examples

PERCOLATION THEORY

Baptiste Mokas

baptiste.mokas@gmail.com

weeki.io/dynamical-système

linktr.ee/baptistemokas

+33 7 69 08 54 19

KNOWLEDGE TREE

$$S(\beta_1, \beta_2) = n\left(\beta_1 - (\bar{y} - \beta_2\bar{x})\right)^2 + \left(\sum_{i=1}^{n}(x_i - \bar{x})^2\right)\left(\beta_2 - \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2}\right)^2$$

$$+ \left(\sum_{i=1}^{n}(y_i - \bar{y})^2\right)\left(1 - \frac{\left(\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})\right)^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2 \sum_{i=1}^{n}(y_i - \bar{y})^2}\right),$$

KEYWORDS (NEW)

KEYWORDS

- Nonlinear Models
- Nonlinearity
- Nonlinear Regression
- Dynamical Systems
- Continuous Systems
- Discrete Systems
- Phase Space
- Trajectories
- Chaos Theory
- Butterfly Effect
- Attractors
- Strange Attractors
- Logistic Map
- Bifurcation Diagrams
- Chaos Emergence
- Fractals
- Self-Similarity
- Fractal Geometry
- Percolation Theory
- Site Percolation
- Bond Percolation
- Percolation Clusters
- Critical Phenomena
- Critical Exponents
- Universality in Percolation
- Scaling Laws
- Network Percolation
- Social Networks
- Epidemic Spread
- Complex Networks
- Porous Media
- Fluid Flow
- Geology
- Hydrology
- Disordered Systems
- Random Resistor Networks
- Ising Model
- Materials Science
- Applications of Nonlinear Models

Use Case with python

Use Case: Modeling Population Growth with the Logistic Map

In the context of the course on Nonlinear Models and Chaos Theory, which covers topics related to nonlinear modeling, chaos theory, percolation theory, and their applications, let's explore a use case related to modeling population growth using the logistic map.

Description:

In this use case, we will apply the logistic map, a classic example of a nonlinear dynamical system, to model the population growth of a hypothetical species. The logistic map exhibits chaotic behavior under certain conditions, making it an interesting system to explore.

Key Components:

Introduction to Nonlinear Models: Understanding the distinction between linear and nonlinear models, the importance of nonlinearity in modeling, and real-world applications of nonlinear models.

Nonlinear Regression: Comparing nonlinear regression to linear regression, formulating nonlinear models, and estimating parameters using nonlinear regression techniques.

Dynamical Systems: Introduction to dynamical systems, differentiation between continuous and discrete dynamical systems, and the concept of phase space and trajectories.

Chaos Theory Basics: Exploring chaos theory, including the butterfly effect, attractors, and strange attractors. Understanding the chaotic behavior of nonlinear systems.

Logistic Map: Examining the logistic map, its equation, and its application in modeling population growth. Analyzing bifurcation diagrams and chaos observed in the logistic map.

Python Code Example (Population Growth Modeling with the Logistic Map):

```python
import numpy as np
import matplotlib.pyplot as plt

# Logistic map function
def logistic_map(r, x):
    return r * x * (1 - x)

# Parameters
r_values = np.linspace(2.4, 4.0, 1000)  # Range of growth rates
x0 = 0.5  # Initial population density
n_iterations = 1000  # Number of iterations

# Plot bifurcation diagram
plt.figure(figsize=(8, 6))
for r in r_values:
    x = np.zeros(n_iterations)
    for i in range(n_iterations):
        x[i] = x0
        x0 = logistic_map(r, x0)
    plt.plot([r] * n_iterations, x, ',k', alpha=0.25)

plt.xlim(2.4, 4.0)
plt.ylim(0, 1)
plt.xlabel('Growth Rate (r)')
plt.ylabel('Population Density')
plt.title('Bifurcation Diagram of Logistic Map')
plt.show()
```

In this code, we define the logistic map function and explore the bifurcation diagram of the logistic map by iterating through different growth rates (r_values). The bifurcation diagram reveals the chaotic behavior of population growth as r increases, with the emergence of multiple bifurcation points and eventually chaotic regions.

This use case illustrates how nonlinear models, such as the logistic map, can capture complex and chaotic phenomena, which are important in fields like ecology and population dynamics.

TOP 20 REFERENCES

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TEXTE DE DESCRIPTION DU COURS

Delve into the captivating world of nonlinear models, where you'll explore the distinctions, applications, and theories of nonlinear modeling.

In the first part, you'll distinguish between linear and nonlinear models, uncover the importance of nonlinearity in various applications, and delve into the intricacies of nonlinear regression, including model formulation and parameter estimation. You'll also embark on an exploration of dynamical systems, understanding both continuous and discrete systems, as well as the concept of phase space and trajectories.

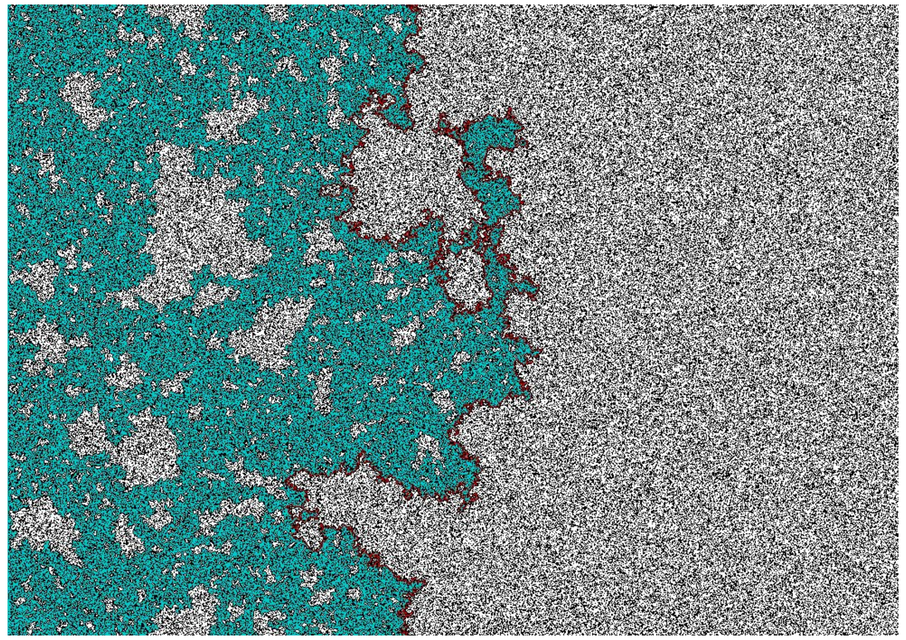
The course then immerses you in the realm of nonlinear models and chaos theory. You'll grasp the fundamentals of chaos theory, including the intriguing "Butterfly Effect" and the concept of attractors, including strange attractors. Dive into the world of the logistic map, uncovering its connection to population growth, bifurcation diagrams, and the emergence of chaos. Additionally, you'll explore fractals and self-similarity, gaining insights into fractal geometry, self-similarity in nature, and various practical applications of fractals.

Part 3 introduces you to percolation theory, starting with an understanding of what percolation theory is and its relevance in networks and materials. You'll delve into percolation models, including site percolation, bond percolation, and percolation clusters. Explore critical phenomena, including critical exponents, universality in percolation, and scaling laws.

The course culminates in Part 4, where you'll discover the real-world applications of percolation theory. Dive into network percolation, exploring its role in social networks, epidemic spread, and the robustness of complex networks. Explore porous media and transport, investigating percolation in porous materials, fluid flow in porous media, and its applications in geology and hydrology. Lastly, delve into disordered systems, including random resistor networks, the Ising model, and its relevance in materials science.

By the end of this course, you'll have a deep understanding of nonlinear models and their diverse applications, from chaos theory to percolation theory, equipping you with valuable knowledge for various fields of study and research.

DESIGN DE LA CARD



Author: Baptiste Mokas, Weeki

Course Name: Simple Linear Regression

#NonlinearModels
#ChaosTheory
#PercolationTheory